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Society for Industrial
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SIAM-NSF Modeling across the Curriculum

Executive Summary

The second Modeling Across the Curriculum Workshop extended the work of the first by using three working groups to

- incorporate explicit work on modeling in and for early grades (pulling early grades work out of the very different high school realm)
- deepen discussion of and refine the action items for developing high school curricular materials and strategies for getting those materials used widely
- develop two explicit pathways to address effective strategies for influencing undergraduate STEM education

All three working groups emphasized the importance of assessment and of working together (among the workshop groups as well as with the mathematical sciences and education communities of experts) to support large-scale transformation in K-16 education.

This Executive Summary will focus on recommendations for what SIAM can accomplish toward the large goals outlined by the working groups. As in MACI, the top level goal of the proposed efforts is to:

ENGAGE AND KEEP YOUNG PEOPLE IN STEM DISCIPLINES, FROM K-12 THROUGH UNDERGRADUATE (AND GRADUATE) STUDIES, AND INTO THE WORKFORCE.

The Recommended Actions from MACI still hold: (1) Expand modeling in K-12; (2) Develop a high school one-semester or one-year modeling course with stratified content; (3) Develop modeling-based undergraduate curricula; and (4) Develop a repository of materials for all aspects and levels of math modeling instruction and understanding.

MACII elaborates on these to include the following Recommended Actions.

1. Early Grades Working Group Recommendations

- a. Produce materials, including classroom posters, videos, materials for teacher training and professional development, released standardized test items and classroom projects that help communicate what mathematical modeling is.

- b. Develop and disseminate (perhaps by joint SIAM/NCTM publication) rich mathematical modeling problems and examples of how to enact them in the early grades classroom.
- c. Create and disseminate videos of college students interviewed by middle school students about REUs and capstones in Industrial and Applied Mathematics.
- d. Create and disseminate videos of teachers interacting with mathematicians; Videos of teachers facilitating modeling; Videos of students modeling.
- e. Create SIAM/ASA websites where teachers can view lesson plans. Include a SIAM library of models that have been vetted by the community and a library of best practices in the teaching and learning of modeling.
- f. Create a nationwide modeling challenge for early grades.
- g. Work with testing agencies to devise mathematical modeling problems and assessment rubrics.
- h. Record video vignettes of someone who is deeply familiar with mathematics talking about modeling with a teacher to help honor both professional behaviors.
- i. Create social networking site for teachers and mathematicians to talk about modeling.
- j. Develop models of peer mentoring. Middle school/High School/Undergrads can mentor students and participate as assistants in professional development for teachers.
- k. Offer a seminar at state-wide math educational conferences on incorporating modeling activities in the classroom.
- l. Create a mapping of topics not traditionally taught by modeling to modeling activities
- m. Think about ways to connect teachers and faculty to math education literature related to mathematical modeling.
- n. Identify those creative teachers who are already in the school and videotape teaching and students explaining what they are doing.

2. High School Working Group Recommendations

- a. GAIMME Report: Inspired by the ASA's GAISE Report, we call for a report outlining Guidelines for Assessment and Instruction in Mathematical Modeling Education (GAIMME).
- b. AIM-Style Workshop: We propose that Katie Fowler (Clarkson University) take the lead in proposing and leading a workshop (possibly at AIM, the American Institute of Mathematics) focused on developing a high school mathematical modeling course and suggesting standards for secondary modeling education.



- c. Infusion Working Group: We propose that a working group of active participants be charged with formalizing strategic approaches to address challenges teachers face in infusing modeling in their daily practice. (This is a curricular and cultural focus.)
- d. Professional Development Working Group: We propose that a working group of active participants be charged with developing recommendations to support teachers as they improve or develop their expertise in mathematical modeling content. (This is a content and practice focus.)
- e. Assessment Working Group: We propose that Rebecca Nichols of ASA and others interested in assessment develop several community charges about assessing the success and health of the Modeling Across the Curriculum enterprise and assessing quality mathematical modeling education activities.
- f. Repository: We recommend a small group develop a proposal for a curated repository of modeling resources, preferably peer-reviewed. (This is likely a huge, ongoing project, needing foundation support for any chance of being successfully and popularly used by teachers and the public.)
- g. Public Awareness: We recommend the professional societies take the lead (e.g., SIAM 2015 MPE-inspired year of modeling with a Math Awareness Month suite of materials about modeling and launching an affiliated Student Innovation in Math Modeling program).

3. Undergraduate Working Group Recommendations

- a. SIAM create an activity group on Applied Math Education. This would provide numerous opportunities for cooperation, collaboration, and recognition.
- b. Generate studies in two main challenge areas. The first is to illustrate how mathematics connects to the rest of the world by identifying its past and current successes and articulating to STEM practitioners, and the public as a whole, the essential and centralizing role that mathematical modeling plays in innovation. The second is to identify and disseminate more targeted strategies for mathematicians to attract and retain students into STEM fields through mathematical modeling.
- c. Improve cooperation across professional organizations: the SIAM Education Committee should continue to establish connections with the education VPs of other societies and organizations. Examples of organizations include MAA, AMS, AAAS, NCTM, ASA, AERA, AMTE, SIAM, IEEE CSS, ACM, COMAP, SCB, SMB, INFORMS, RUME, AMTYC, APS, CBMS, MSO, AMTA, CSEE, to name a few. It would also be helpful to connect to centers that are modeling-friendly such as DIMACS, Cause, etc.

As of January 1, 2015, the recommendation to form a SIAM Special Interest Group on Applied Mathematics Education has been implemented, and the writers of this report hope that the SIAG-ED will take on many of the challenging tasks outlined by the working groups.



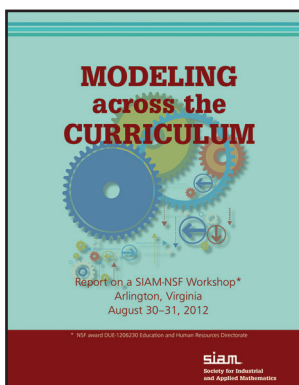
Introduction and Background

The Society for Industrial and Applied Mathematics, SIAM, was awarded a second National Science Foundation grant to continue the work on increasing mathematical modeling and computational applied mathematics in high school and college curricula, and to add a thread considering the implications and possibilities in the early grades. Both workshops grew out of discussions between SIAM and NSF Education and Human Resources representatives early in 2011 on the topics of undergraduate and K-12 courses and programs, college readiness and career preparation.

The main themes of the second workshop, aptly titled, 'Modeling across the Curriculum II,' MaC II, investigated ways to increase mathematical modeling across undergraduate curricula and to develop modeling content in the K-12 educational arena. Within this context it was also important to assess college STEM readiness.

The overarching goal for both workshops and the work that results from them is to:

Engage and Keep Young People in STEM Disciplines, from K12 through Undergraduate (and Graduate) Studies, and into the Workforce.



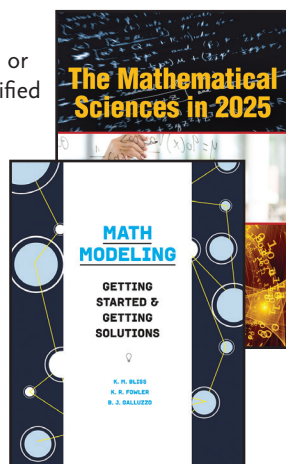
This objective is simply stated, but less simply achieved. Developing and implementing strategies for achieving that objective are fundamental initial steps.

The MaC II workshop picked up where the first workshop left off in many respects. The report from MaC I [1] is available online at www.siam.org/reports/modeling_12.pdf. The major recommendations from the first workshop can be categorized as fitting four different categories:

- Expand modeling in K-12
- Develop a high school one semester, or one year modeling course (with stratified content)
- Develop modeling-based undergraduate curricula
- Develop a repository of materials for math modeling instruction and understanding.

For MaC II the evaluation theme of MaC I became an implicit requirement of all strands.

One outcome that supports the first of these recommendations was the handbook Math Modeling: Getting Started and Getting Solutions [2] which was produced by SIAM as a cooperative venture between the MaC initiative and the Moody's Mega Math Challenge, M3, which is organized by SIAM on behalf of the Moody's Foundation, m3challenge.siam.org/.



While the MaC workshops are relevant at a time of growing concern about America's standards in math and science education, they were especially timely in the wake of the undergraduate STEM education report Engage to Excel: Producing One Million Additional College Graduates with Degrees in Science, Technology, Engineering, and Mathematics [6] released by the President's Council of Advisors on Science and Technology (PCAST) in February 2012. The widespread adoption of the Common Core State Standards in Mathematics [5] adds further urgency to these deliberations.

The objectives of the workshop addressed several key issues raised both in the PCAST report, such as increasing student preparedness for STEM majors and overall enhancement of STEM education in the first two years of college, and in the influential National Academies report, The Mathematical Sciences in 2025 [3]. The results of the discussions should also help in responding to criticisms of the implementation of the Common Core State Standards and especially the recommendations to increase modeling and application-based learning in school curricula.

Mathematical modeling has the potential to increase interactions and interconnections between various STEM areas. The PCAST report, CCSSI recommendations, and anecdotal information from high school and college educators call for a more coordinated approach to STEM education. The MaC workshops have begun the process of evaluating and developing material to enhance the STEM educational spectrum in a coordinated manner. Through mathematical modeling, students in K-12 can prepare for STEM college majors and careers, thus increasing the pipeline of scientific and technical talent in America. Topical coverage should be broad in terms of both content and audience. Applied and Computational Mathematics including Statistics (ACMS) is a natural topical center for coordinated STEM programs both feeding and gaining from all other STEM fields. It should also be noted, that this preparation also serves majors in social, financial and life science majors well. This was an added theme of the recent CBMS Forum in October 2014, <http://www.cbmsweb.org/Forum5/index.htm>.

A number of curriculum options were explored in MaC I and further developed in MaC II. One possible avenue is the development of undergraduate STEM degree programs as alternatives to traditional discipline majors. These might mirror the growth of Computational Science and Engineering programs over the past 10 – 15 years, and are likely to be reflected in the growth of Data-Enabled Science and Engineering in the next several years. A key question is the extent to which mathematical modeling is treated as a stand-alone "course" or whether it should be integrated as the Modeling across



the Curriculum title suggests. Coordinating the fundamental mathematics, computation, statistics and science content to support application in a wide range of STEM fields may have strong appeal to potential students.

The full agenda for the 2.5 day workshop is included as Appendix A. The first afternoon and early evening were plenary sessions, including an introduction from Joan Ferrini-Mundy, Assistant Director of the National Science Foundation, Education and Human Resources Division. As is noted in the undergraduate section of this report, Dr. Ferrini Mundy challenged us “to think about effective ways to educate students at the crossroads of modeling, data science, information science, computational science, and computational thinking.” This is entirely consistent with the remarks in the previous paragraph.

There followed a general introduction with some background and summary of the first workshop in order to establish our starting point for MaC II discussions. This objective was furthered through a panel discussion among the three theme leaders (Humpherys, Levy and Socha, moderated by Turner). Topics included the following points:

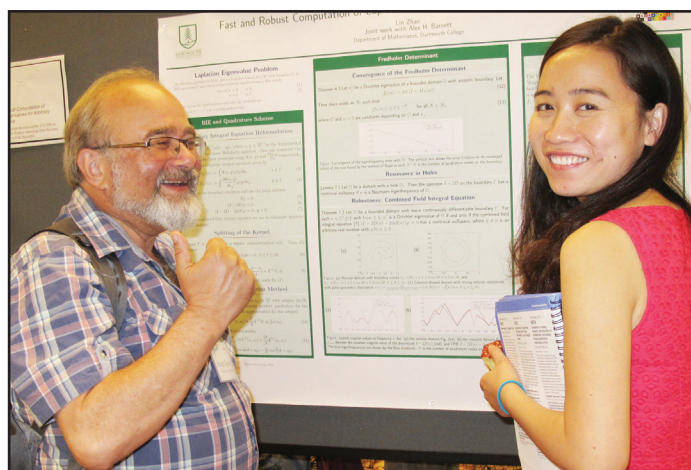
- We spent a lot of time in MaC I discussing the definition of modeling. What were some of the issues and outcomes of that discussion?
- What are the differences between having a stand-alone modeling course and infusing modeling into the mathematics of different courses at different level? Can we create materials that would be flexible enough for both?
- What are our goals in trying to break up our discussions by grade level?
- What is the role of algorithms in modeling?
- What would modeling look like to a third grade class?
- What are the issues with assessment of mathematical modeling?
- What can be the role of NCTM, SIAM, ASA and other organizations in these efforts?
- The goal of this workshop is to generate not just a report, but a set of action items that a set of us will work on for the next few years. We will generate ideas for proposals related to programs, materials and training. What are potential audiences for the mini-proposals that we hope to generate from this meeting?

The afternoon plenary session included two other presentations. One was a keynote address from Mark Green on the Mathematical Sciences in 2025 report in which he highlighted many of the common themes of that report and the Modeling across the Curriculum goals. The second was a presentation on Mathematical Modeling: Getting Started and Getting Solutions by two of its authors, Katie Fowler and Ben Galluzzo.

Overview of the Working Group Reports

Modeling in the Early Grades

The second Modeling across the Curriculum Workshop was probably the first time a SIAM-organized meeting paid any real attention to mathematical modeling in the early grades, K-6. This new focus recognizes that improving the output from the mathematical pipeline requires attention to the entire educational process. We want to form partnerships with administrators, teacher educators,



mathematics education researchers, teachers and modeling contest coaches, to discover how best to teach mathematical modeling in the early grades and to determine what will work in the classroom.

There are several motivations for incorporating mathematical modeling in the curriculum from the earliest stages. For example the Jasper study [4], [5] found that “students who worked on real-world problems demonstrated less anxiety toward mathematics, more likely to see math as relevant to real life, more likely to see it as useful, more likely to appreciate complex challenges.” The study also found a positive effect for both previously high and low achieving students.

Even at the undergraduate level, instruction is mostly comprised of teaching models and applications, but there is little opportunity for students to experience the creative aspects of the modeling process. Undergraduate students might only be charged with independently developing mathematical models in a course titled “Modeling” or in industrial math projects rather than across all STEM courses. Thus instructors at all grades, kindergarten through university are facing the same challenge of training students to be modelers. There is a body of literature, but no consensus in the United States on strategies or comprehensive programs to develop modeling capabilities in students. This is true despite the widely held view that modeling acts as an excellent motivator for interest, and consequently for ability, in the mathematical sciences.

One outcome of the first MaC workshop was that participants questioned whether a single high school course would be sufficient to prepare students for undergraduate studies in the STEM fields. Participants concluded that modeling should instead be taught in every grade level, either through infusion into the curriculum, or by constituting the very backbone of the curriculum. As a result, one task in the MaC II workshop was to determine how modeling might successfully be integrated into the early grades (K-6) curriculum.

The early grades chapter of this report describes plans for future efforts emphasizing both public relations (communicating to everyone what mathematical modeling is) and professional development for teachers. Proposed ideas include a national center for Mathematical Modeling and Industrial and Applied Mathematics Education, which would support the entire mathematics educational spectrum with activities similar to the existing mathematical institutes, with additional professional development.

High School Modeling Courses

The recommendations from MaC I included two major thrusts related to the K-12 environment:

- Expanding modeling in K-12, and
- Development of a high school one semester, or one year, modeling course (with stratified content)

The inclusion of the early grades discussion in MaC II was in large degree a consequence of the first recommendation, and both were important stepping off points for discussions in the High School group.

The High School Working Group developed seven recommendations aimed at influencing content and teaching practice at the high school level. The group encourages careful consideration of the current education climate: the mathematical sciences community needs to convince policy makers, test makers, school leaders, teachers, and parents that the results of infusing mathematical modeling throughout the high school curriculum will be well worth the investment of time. A basic premise for much of the discussion is that mathematical modeling embodies mathematical thinking – modeling should not be tied to calculus or statistics but should be infused in all kinds of quantitative courses across the high school curriculum. Mathematical modeling should also form the heart of some sort of capstone or aspirational course for high school students.

Both the infusion approach across the curriculum and development of an aspirational or capstone course are strongly encouraged because these parallel efforts are likely to reach different groups of students. The specific recommendations include establishing working groups to examine the infusion model and to initiate a workshop, perhaps in the American Institute for Mathematics (AIM) style or even under their auspices, specifically to develop a high school course. The success of the Statistics community in expanding statistical content in the K-12 curriculum through the ASA's Guidelines for Assessment and Instruction in Statistics Education (GAISE) report [6] was seen as a model to emulate with a comparable effort in mathematical modeling education.

Additional recommendations concern establishing continuing working groups to develop professional development models, assessment materials, and a curated repository of good materials for modeling education at all levels. The final recommendation probably applies to all three themes: there should be a strong public awareness campaign to educate the wider population on both what mathematical modeling is and why it is important to the development of successful STEM educational programs and the desired future technically able workforce.



Modeling across the curriculum in undergraduate STEM degree programs

The working group on the undergraduate curriculum focused on similar themes. Because the conference began with a challenge from opening speaker Joan Ferrini-Mundy of the National Science Foundation, the group decided to think about effective ways to educate students at the crossroads of modeling, data science, information science, computational science, and computational thinking. After much discussion, the group identified two main pathways to help meet this goal.

The first recommended pathway is to commission two reports to inform and educate stakeholders on the central role that mathematical modeling plays in society. The first report would illustrate how mathematics connects to the rest of the world by identifying its past and current successes. The report would articulate to STEM practitioners, and the public as a whole, the essential and centralizing role that mathematical modeling plays in innovation. The second report would identify and disseminate targeted strategies for mathematicians to attract and retain students in STEM fields through mathematical modeling at all stages of the undergraduate experience.

The second pathway was to have SIAM and other professional organizations play a greater role in creating and supporting communities of practitioners in applied mathematics education.

The group recommended that a SIAM Special Interest Activity Group (SIAG) on Applied Mathematics Education be formed in order to provide opportunities for cooperation, collaboration, and recognition. Examples include conferences, sessions at the annual meeting, email lists, SIAM-backed blogging, and even perhaps an online magazine. There could also be awards given to departments and individuals recognizing their contributions.

Following the workshop a proposal for such a SIAG was developed and submitted to SIAM. It was favorably received and approved by SIAM Council and Board of Trustees in

July 2014 to begin operations in January 2015.

The Undergraduate group also identified a need for cooperation across professional organizations and it was recommended that the SIAM Education Committee continue to make and establish connections with the education VPs of other societies and organizations. Such connections generate opportunities to address important questions and try to get some consensus around educational issues in the broader STEM community. Several specific questions are posed and discussed in the undergraduate group chapter of this report.

Modeling across the Curriculum Early Grades Report

Rachel Levy, Laura Pahler, Michelle Cirillo, John Pelesko, Matt Ellinger, Padmanabhan Seshaiyer, Stacy Brown

I. Overview

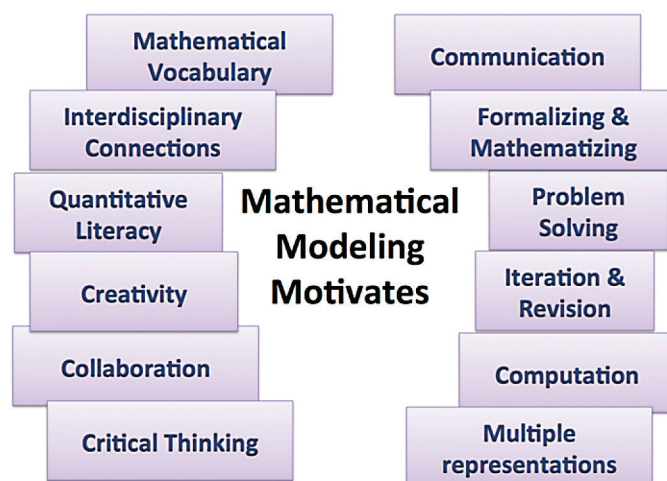
The second Modeling across the Curriculum Workshop was one of first SIAM-organized meetings with a working group focused on mathematical modeling in K-6. Our group forged new collaborations among applied mathematicians, mathematics education researchers, teacher educators, teachers and instructional designers. Our goal was to investigate and develop best practices for teaching mathematical modeling in the early grades and to determine what will work in the classroom. In this early effort the team recognized that we would not have all the answers, and would need everyone's input and partnership.

There are several motivations for incorporating mathematical modeling in the early grades curriculum. For example the Jasper study (1992, 1997) found that "Students who worked on real-world problem demonstrated less anxiety toward mathematics, more likely to see math as relevant to real life, more likely to see it as useful, more likely to appreciate complex challenges." The study also found a positive effect for both previously high and low achieving students.

Even at the undergraduate level, instruction is mostly comprised of teaching models and applications, but there is little opportunity for students to experience the creative aspects of the modeling process. Undergraduate students might only be charged with independently developing mathematical models in a course titled "Modeling" or in industrial math projects rather than across all STEM courses. Thus instructors at all grades, kindergarten through university are facing the same challenge of training students to be modelers. There is a body of literature, but no consensus in the United States on strategies or comprehensive programs to develop modeling capabilities in students. This is true despite the widely held view that modeling acts as an excellent motivator for interest, and consequently for ability, in the mathematical sciences.

One outcome of the first MaC workshop was that participants questioned whether a single high school course would be sufficient to prepare students for undergraduate studies in the STEM fields. Participants concluded that modeling should instead be taught in every grade level, either through infusion into the curriculum, or by constituting the very backbone of the curriculum. As a result, one task in the MaC II workshop was to determine how modeling might successfully be integrated into the early grades (K-6) curriculum.

The early grades report describes plans for future efforts emphasizing both public relations (communicating to everyone what mathematical modeling is) and professional development for teachers. We also propose a national center for Mathematical Modeling and Industrial and Applied Mathematics Education which would serve teachers, math teacher educators, mathematics education researchers, curriculum developers, assessment specialists in addition to mathematicians with similar activities to the existing mathematical institutes with additional professional development.



Recommendations

Many of these recommendations are expanded upon as Action Items below and in the Appendices.

1. Produce materials, including classroom posters, videos, materials for teacher training and professional development, released standardized test items and classroom projects that help communicate what mathematical modeling is.
2. Develop and disseminate (perhaps by joint SIAM/NCTM publication) rich mathematical modeling problems and examples of how to enact them in the early grades classroom.
3. Create and disseminate videos of college students interviewed by middle school students about REUs and capstones in Industrial and Applied Mathematics.
4. Create and disseminate videos of teachers interacting with mathematicians; Videos of teachers facilitating modeling; Videos of students modeling.
5. Create SIAM/ASA websites where teachers can view lesson plans. Include a SIAM library of models that have been vetted by the community and a library of best practices in the teaching and learning of modeling.
6. Create a nationwide modeling challenge for early grades.

7. Work with testing agencies to devise mathematical modeling problems and assessment rubrics.
8. Record video vignettes of someone who is deeply familiar with mathematics talking about modeling with a teacher to help honor both professional behaviors.
9. Create social networking site for teachers and mathematicians to talk about modeling.
10. Develop models of peer mentoring. Middle school/High School/9. Undergrads can mentor students and participate as assistants in professional development for teachers.
11. Offer a seminar at state-wide math educational conferences on incorporating modeling activities in the classroom.
12. Create a mapping of topics not traditionally taught by modeling to modeling activities
13. Think about ways to connect teachers and faculty to math education literature related to mathematical modeling.
14. Identify those creative teachers who are already in the school and videotape teaching and students explaining what they are doing.

II. Advantages of Teaching Mathematical Modeling in K-6

While applying mathematics to real world applications can have advantages for learners of all ages, there may be advantages specific to introducing modeling early.

- Thinking creatively may come more easily to children first learning and exploring mathematical concepts.
- Young students have high potential to become fluent – native speakers, thinkers and dreamers of mathematics.
- Kindergarten students have been shown to be able to use manipulatives to independently solve traditional multiplication or division problems (a 3rd or 4th grade standard) they have never seen before. (Carpenter, et al, 1993) which is evidence that young students bring knowledge to the classroom --we don't have to wait to incorporate modeling activities until we have "shown them how" to do everything.
- Teachers can lay the groundwork for mathematical modeling through pre-modeling activities, such as making simplifying assumptions about a situation or modeling mathematics (rather than mathematical modeling).
- Students can be coached to use trial and error to approach problems where they have not been shown the solution approach.
- Because early grades teachers are generalists, they can address several subjects simultaneously through modeling activities.
- Teachers may have the flexibility to seize on a moment where modeling can happen throughout the curriculum. Students can also learn to recognize these times. For example, the class could graph the "happiness" over time of a particular character in a story.



III. Common Core State Standards and Mathematical Modeling

The Common Core State Standards for Mathematics [1], released in 2010, includes eight Standards for Mathematical Practice, which describe "processes and proficiencies" that mathematics educators at all levels should seek to develop in their students.

Mathematical Modeling has a privileged place in the CCSSM. It is the only topic that is both a practice and a content standard. Mathematical modeling is also the only mathematics standard that is also a science standard. Thus, mathematical modeling has been elevated to a new level in CCSSM.

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

While model with mathematics has its own category, it is also easy to see how practicing mathematical modeling can strengthen students' proficiency in the other practices.

Common Core by Domain K-8

	K	1	2	3	4	5	6	7	8
Counting and Cardinality	X								
Operations and Algebraic Thinking	X	X	X	X	X	X			
Number and Operations in Base Ten	X	X	X	X	X	X			
Numbers and Operations-Fractions				X	X	X			
Measurement and Data	X	X	X	X	X	X			
Geometry	X	X	X	X	X	X	X	X	X
Ratios and Proportional Relationships							X	X	
The Number System							X	X	X
Expressions and Equations							X	X	X
Statistics and Probability							X	X	X
Functions									X

Horizontal and vertical integration: We would like to increase teachers' awareness of how modeling is implemented both horizontally (across curriculum at their level) and vertically (where they are going and what they are coming from).

IV. Misconceptions about Mathematical Modeling

A 2013 PhD Thesis by Heather Gould at Columbia aimed to determine the conceptions and misconceptions held by teachers about mathematical models and modeling in order to aid in the development of teacher education and professional development programs through four research questions:

- 1) How do teachers describe a mathematical model?
- 2) How do teachers describe the mathematical modeling process?
- 3) What do teachers believe to be the purpose of mathematical modeling?
- 4) What are the misconceptions evident in the teachers' descriptions of mathematical models and the mathematical modeling process?

Most teachers correctly understood that: (a) Mathematical models can be equations or formulas or example, a quadratic equation or $d = rt$, the distance-rate formula. (b) Mathematical models can be used to explain the underlying causes in a given situation. (c) The mathematical modeling process involves determining if a solution makes sense in terms of the original situation and (d) Repeating steps and making revisions may or may not be part of the mathematical modeling process.

But a majority of the teachers held the following misconceptions: (a) Mathematical models can be physical manipulatives or example, fraction tiles, pattern blocks, or three-dimensional solids (like cubes, octahedra, and other polyhedra). (b) Mathematical modeling situations come from "whimsical" or unrealistic scenarios. (c) The mathematical modeling process always results in an exact answer or exact answers. The teachers did not realize that the

mathematical modeling process necessarily involves making choices and assumptions. While this report focuses on the response of the majority, if even a tenth or quarter of all teachers carry a misconception, many students will likely be affected.

ACTION ITEM: Produce materials, including classroom posters, videos, materials for teacher training and professional development, released standardized test items and classroom projects that help communicate what mathematical modeling is.

V. Teacher Preparation and Professional Development

Teacher Preparation

Elementary grades teachers generally have very different preparation in mathematics than middle and high school teachers. They are unlikely to have been mathematics majors in college and may have had few or no courses to develop their content knowledge of early grades mathematics (and beyond). They generally have had courses in pedagogy and have passed examinations that test some basic mathematics skills up to 8th grade mathematics (but do not necessarily have an understanding of why a solution algorithm works or why concepts are true). For certification tests with questions from multiple subjects, it may be possible to pass by being strong in some areas though weak in others. While many elementary teachers are enthusiastic about mathematics, some describe themselves as "math phobic" and may have chosen to teach lower grades to avoid higher mathematics. However, studies show that the opportunity to deepen their own content knowledge not only improves the teachers' competence and confidence, it directly impacts the competence of their students. (Brown).

In their preparation, teachers may not have been exposed to mathematical modeling and furthermore might use the term

“modeling” in different ways. For example, one of the phases in a lesson plan is called modeling. For example, if the elements of a lesson plan may include: Anticipatory set; Learning Objective and Purpose; Input (the lesson); **Modeling**; Checking for Understanding; Guided Practice; Independent Practice; Closure (cite Madeline Hunter).

These elements can appear in any subject, including mathematics. In this type of plan, model usually means students watch the teacher work a problem on the board and perhaps take notes. It could also be used to describe the teacher modeling thought processes using “think-alouds.” In a think-aloud, while solving a problem, the teacher might say things like “does my answer make sense?”, “Let me check back with the problem to make sure I answered the question being asked”, and “How could I estimate this number?” This way the teacher is modeling mathematical problem-solving.

In addition, teachers may “model mathematics” using manipulatives or multiple representations, such as providing models of multiplication by showing repeated addition. The book *Children’s Mathematics: Cognitively Guided Instruction* (Carpenter, et al 1999) uses the term “Direct Modeling” to mean using manipulatives (or drawings) in one-to-one correspondence to represent objects. For example, kindergarteners might use 5 blocks or draw 5 sticks to represent five fish.

To show how rare is elementary teachers’ professional exposure to mathematical modeling: a search using the term “modeling” in LearnZillion.com lessons yields 259 results, most of which involve models but not the modeling process.

Thus when we communicate with teachers, we must acknowledge that they may commonly use the term modeling differently than the way applied and industrial mathematicians use it. We might suggest that in some of these non-modeling cases (such as using manipulatives), teachers use the word “represent” rather than model.

We want to emphasize that we do not want to approach teacher education from a “deficit model.” Even though teachers are under

a lot of pressure and strapped for resources (especially time), many are eager to learn new mathematical ideas and ways of sharing them with students. We can celebrate the perspective of teachers, who can help us see ideas in new ways. For example, the visuals generated by a teacher who worked on a “locker problem” in a professional development context has been widely disseminated and used by other teachers (Seshaiyer et al).

To help teachers facilitate mathematical modeling in their classroom, we might propose the following teaching model based on Smith and Stein’s (2011) five steps for productive classroom discussions for sharing student work.

Anticipate – think about which strategies students will use

Monitor – which strategies students actually use

Sequence – choose the order in which students present from most pictorial to most abstract, or least efficient to most efficient

Schedule – ask students to communicate in a certain order

Connect – back to the mathematical target.

Teachers may guide discussions using the following six “moves” to elicit continued thinking and reasoning from students (Cirillo 2013, and Herbel-Eisenmann, Steele, & Cirillo, in press):

- Waiting (e.g., Can you put your hands down and give everyone a minute to think?)
- Inviting Student Participation (e.g., Let’s hear what kinds of conjectures people wrote.)
- Revoicing (e.g., So what I think I hear you saying is that if there was only one point of intersection, it would have to be at the vertex. Have I got that right?)
- Asking Students to Revoice (e.g., Okay, can someone else say in their own words what they think Emma just said about the sum of two odd numbers?)

- Probing a Student’s Thinking (e.g., Can you say more about how you decided that?)
- Creating Opportunities to Engage with Another’s Reasoning (e.g., So what I’d like you to do now is use Nina’s strategy to solve this other problem with a twelve-by-twelve grid.)
- http://www.nctm.org/uploadedFiles/Research_News_and_Advocacy/Research/Clips_and_Briefs/research%20brief%2020%20-%20strategies%20%20discussion.pdf

ACTION ITEM: Develop professional development programs that train teachers (and perhaps also math specialists, district leaders, mathematicians and parents) how to do mathematical modeling and facilitate mathematical modeling for early grades.



Professional Development in Mathematical Modeling

We hope that an outcome of this workshop will be to develop professional development activities for early grades teachers in mathematical modeling. Math Teacher Circles and Mathematics and Science Partnerships (State and National) provide examples of successful professional development, usually with the following elements:

Elements of Successful Professional Development

- Multiple meetings over a long period
- Lesson Creation and Lesson Study
- Reflection tasks
- Pre and post Assessment
- Dissemination

ACTION ITEM: Record video vignettes of someone who is deeply familiar with mathematics talking about modeling with a practitioner (teacher) to help honor both professional behaviors. The teacher can help the mathematician see something from their point of view. Honor those occasions. Offer the video through Edutopia or NCTM. Work to break down fear and cultural barriers.

ACTION ITEM: Create social networking site for teachers and mathematicians to talk about modeling. Build community.

ACTION ITEM: Develop models of peer mentoring. Middle school/High School/Undergrads can mentor students and participate as assistants in professional development for teachers. High School students who have done Moody's can coach elementary kids. Undergraduates who do MCM/ICM can help coach Moody's.

ACTION ITEM: Offer a seminar at state-wide math educational conferences (e.g. the CMC-South and -North conferences) on incorporating modeling activities in the classroom. These conferences often represent the cutting edge in trends and research in education, and many math coaches and teacher leaders attend.

ACTION ITEM: Create a mapping of topics not traditionally taught by modeling to modeling activities

ACTION ITEM: Think about ways to connect teachers and faculty to math education literature related to mathematical modeling. Perhaps include this as part of a repository.

ACTION ITEM: Identify those creative teachers who are already in the school – videotaping teaching and students explaining what they are doing. Students should collect this as their portfolio of problem-solving. Teachers do not have time to observe each other. Get somebody willing to come videotape the teacher setting up an activity that works in the classroom. Need to involve IRB but it can be done. Exposure should be a piece of PD. Conversations about what happened.

Use and Definition of the term “modeling”

We use the progression developed by the High School working group to help communicate what we mean by modeling. It can be helpful to look at a single problem to see that how teachers and students engage the problem determines whether or not the students practice modeling. For example, you can consider the following progression:

- **Bare mathematics:** Given two points, find the line function. This manner of presenting a problem is found in many texts and online resources. The problem is often found after a section explaining how to work that type of problem and within a problem set with many similar tasks.
- **Applications:** If movie tickets are \$9 and you start with \$50 for making change, what is the linear function of how much money in the cash register? These word-problem-type tasks are often found as the last problems in a chapter and some online resources, especially some of the new ones aligned with the CCSS.
- **Models:** You are the cashier at a cinema, what is the linear function that represents how much is in the cash register? This type of problem provides a way to start to teach what modeling is by presenting the model and the relevant situation. But the student is not yet making assumptions or choosing/developing a solution approach.
- **Mathematical Modeling:** You are running a movie theater, how should you monitor the amount of cash you have? This problem represents the real world in that there are multiple ways to approach the problem, and no clear direction on what

information to include and how to use it.

At all levels, there can be a tension between (a) problem-based iterative modeling with multiple possible solutions and techniques and (b) models that motivate a pre-determined mathematical technique. Teachers may need students to practice a particular technique and may want to introduce a real-life situation to motivate the use of that technique. While this lies in the applications or models realm, the task would still be closer to modeling than “bare mathematics.”



Teachers can learn how to take a “bare” mathematics problem and make it more like a modeling problem. For example, instead of giving some scores and asking students to average them, you could give an average score and ask students to discuss what the possible individual scores could have been. One way to develop these questions is to think of the version where there is only one answer and then devise a similar question where there are multiple answers. Sometimes this might just involve saying “this is the answer, what was the question?”



If a teacher decides to assign mathematical modeling tasks, in order for the students to experience mathematical modeling, the teacher must be careful not to reveal too much and consequently **reduce the cognitive demand of the problem.**

We want to encourage **learning by doing, decision-making and creative problem solving** rather than answer-getting. Teachers need to be solid in their mathematics content knowledge so that they can be facile with (1) multiple representations, (2) high cognitive demand and rich tasks, (3) 21st century skills. (4) improving pedagogy through lesson study. Teachers need to be able to work with problems for which there are multiple possible correct answers, including modeling problems for which students need to make both assumptions and decisions.

In some sense, we are reversing the philosophy: Instead of saying “here is the mathematics – solve the problem,” we want teachers to say “here is the problem – find the mathematics to solve it.” We want teachers and students to appreciate the power of failure and iteration the same way many engineers do.

Pollak (2012) reminds us:

The heart of mathematical modeling, as we have seen, is problem formulating before problem solving. So often in mathematics, we say ‘prove the following theorem’ or ‘solve the following problem’. When we start at this point, we are ignoring the fact that finding the theorem or the right problem was a large part of the battle. By emphasizing problem finding, mathematical modeling brings back to mathematics education this aspect of our subject, and greatly reinforces the unity of the total mathematical experience. (p. xi, emphasis added)

ACTION ITEM: Develop and disseminate (perhaps by joint SIAM/NCTM publication) rich mathematical modeling problems and examples (perhaps video) of how to enact them in the early grades classroom.

ACTION ITEM: Create and disseminate videos of college students interviewed by middle school students about REUs and capstones in Industrial and Applied Mathematics.

ACTION ITEM: Create and disseminate videos of teachers interacting with mathematicians; Videos of teachers facilitating modeling; Videos of students modeling. Targets: fluency, culture and content.

Social Justice, Access, and Equity

If students are getting tracked early, then giving them a good start could be critical to their future path. Study on how modeling might benefit all students but especially benefit traditionally underrepresented students.

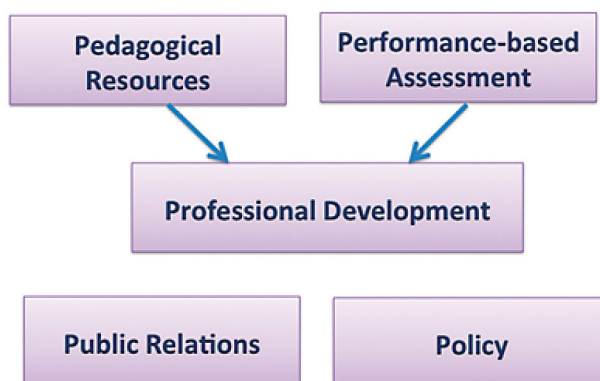
Another important equity issue is access to training in mathematics for early grades teachers. Not only higher mathematics, but deep knowledge of early grades content (CITE Ma?).

There are also ways to raise social justice issues in the classroom. For example, in his 12th grade “math for social justice” class, Eric (Rico) Gutstein engaged students with real problems, such as the number of liquor stores versus grocery stores in different neighborhoods,

http://www.rethinkingschools.org/archive/27_03/27_03_gutstein.shtml .

VI. Future Directions

Mathematical Modeling Priorities



A subset of the early grades working group (Levy, Pahler, Seshaiyer) along with Beth Burroughs submitted a successful NSF STEM-C proposal and have initiated the IMMERSION program, a collaboration between Fairfax County Virginia, Bozeman Montana, and Pomona California schools with George Mason University, Montana State University and Harvey Mudd College. The project will address many of the action items from the working group including curriculum development and repository, professional development and connections to testing organizations.



High School Working Group

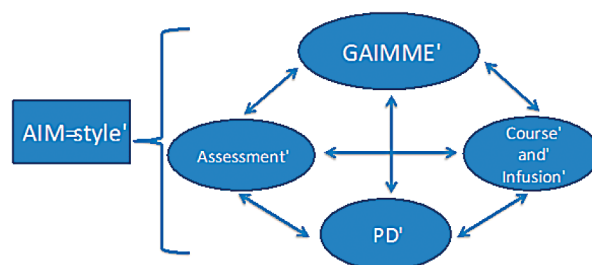
Katherine Socha, Michelle Montgomery

The High School Working Group developed seven recommendations for influencing content and teaching practice at the high school level. We encourage careful consideration of the current education climate: the mathematical sciences community needs to convince policy makers, test makers, school leaders, teachers, and parents that the results of infusing mathematical modeling throughout the high school curriculum will be well worth the investment of time. We believe mathematical modeling embodies mathematical thinking – modeling should not be tied to calculus or statistics but should be infused in all kinds of quantitative courses across the high school curriculum. Mathematical modeling should also form the heart of some sort of capstone or aspirational course for high school students. We strongly urge both infusion across the curriculum as well as development of an aspirational or capstone course, because these parallel efforts are likely to reach different groups of students.

Recommendations

1. **GAIMME Report:** Inspired by the ASA's GAISE Report, we call for a report outlining Guidelines for Assessment and Instruction in Mathematical Modeling Education (GAIMME).
2. **AIM-Style Workshop:** We propose that Katie Fowler (Clarkson University) take the lead in proposing and leading a workshop (possibly at AIM, the American Institute of Mathematics) focused on developing a high school mathematical modeling course and suggesting standards for secondary modeling education.
3. **Infusion Working Group:** We propose that a working group of active participants be charged with formalizing strategic approaches to address challenges teachers face in infusing modeling in their daily practice. (This is a curricular and cultural focus.)
4. **Professional Development Working Group:** We propose that a working group of active participants be charged with developing recommendations to support teachers as they improve or develop their expertise in mathematical modeling content. (This is a content and practice focus.)

5. **Assessment Working Group:** We propose that Rebecca Nichols of ASA and others interested in assessment develop several community charges about assessing the success and health of the Modeling Across the Curriculum enterprise and assessing quality mathematical modeling education activities.
6. **Repository:** We recommend a small group develop a proposal for a curated repository of modeling resources, preferably peer-reviewed. (This is likely a huge, ongoing project, needing foundation support for any chance of being successfully and popularly used by teachers and the public.)
7. **Public Awareness:** We recommend the professional societies take the lead (e.g., SIAM 2015 MPE-inspired year of modeling with a Math Awareness Month suite of materials about modeling and launching an affiliated Student Innovation in Math Modeling program).



Details

GAIMME: The mathematics community, to the best of our knowledge, has not generated any report similar in scope and influence to the GAISE report. Most citizens and many teachers do not know what math modeling looks like, so the mathematical sciences community must provide exemplars of modeling practice and assessment in the curriculum. A first step is to lay out a careful discussion of Guidelines for Assessment and Instruction in Mathematical Modeling Education (GAIMME). A GAIMME report can

- communicate with parents, guardians, counselors, school leaders, and teachers the structure of mathematical modeling for understanding of interdisciplinary problems and its importance to success in STEM studies,
- promote meaningful mathematical literacy for the 21st century,
- make connections to real world experiences and careers,
- reinforce the 4 classroom "C"s in the context of modeling by providing guidance for communication, collaboration, critical thinking, and creativity,
- describe in a broad sense what modeling looks like to develop a shared cultural understanding – that is define the modeling PROCESS, and
- set the stage for assessment development, curriculum development and professional development.

In the current K-16 education climate, where people create a wide variety of modeling courses and activities nationwide, such a report can provide direction for the modeling community to work in concert for the good of students in their studies and careers. The GAIMME writing team should consider carefully the value of promoting education research or evaluation practices that can help validate the long-term effects of mathematical modeling courses or experiences on students and their growth. We should take care that a modeling course does not become a kind of default for weak students to avoid calculus; in fact, it might be a way to support students in strengthening weak backgrounds. The writing team should also take the long view: what can be created now so that in a decade we will see national shifts in education practice. Can universities evolve so that calculus is not the gold standard course for collegiate admission? If so, then having large state universities clearly communicate their values to counselors and leaders of their feeder schools can have a huge impact on those schools adopting mathematical modeling practices as an explicit part of their curriculum. In that context, then, school district leaders who typically create the curriculum maps for departments will be encouraged to make time for modeling in the mathematical sciences at both the middle and high school levels.

The GAIMME report should focus on how modeling should be taught pre-K through 12, and there needs to be close attention to the Common Core Standards with discussion of what modeling looks like at each level. There also should be a substantive discussion of assessment, possibly even with examples of assessment questions.

The major mathematical societies should put pressure on the community to develop and support a plan to assess and improve mathematical sciences education. They should form a panel of professionals with substantive, senior-level experience in education, assessment, and modeling and of professionals who work day-to-day as classroom mathematical sciences educators and supervisors to prepare the GAIMME report. Such a report needs deep knowledge of high school as well as elementary/middle grades and college mathematical sciences and have sub-committees for each major level. There must be practicing classroom teachers in the group. Here are a few professionals whom various members of our working group think would be fantastic: Dick Scheaffer (University of Florida); Henry Pollak (Columbia Teachers College); Joe Skufca (Clarkson University); Zalman Usiskin (Chicago); Rachel Levy (Harvey Mudd College); Dan

Teague (NCSSM); Sol Garfunkel (COMAP); Ben Childers (AZ High school); Greta Mills (NH & FL high school); Al Schoenfeld (Berkeley); Jo Boaler (Stanford); Glenda Lappan (Michigan State); Chris Franklin (Georgia); Darrin Starnes (Lawrenceville School, Princeton); Ellen Mandinach (national level assessment); George Richardson (SUNY Albany); and Joe Malkevitch (CUNY). A full list can be quickly developed for potential participants.

We know that a GAIMME project requires support, and that means finding the right entities to support the writing and promote the result to all sorts of audiences. A lot of details need to be worked out – for example, perhaps there should be some sort of steering committee with representation from the major mathematical sciences and education societies, such as MAA, SIAM, ASA, INFORMS, NCTM, AMATYC, and more. There should be a good review process.

Ultimately, we need this report to be first and fast, or other non-mathematical sciences entities will define the trajectory of mathematical modeling education (possibly through the development of standardized testing policies and test items). Ideally, we would see some proposed methodology to transform the mathematics education community and culture at a large scale through infusing and incorporating mathematical modeling across the curriculum at all levels of the formal education system.

It may be helpful to ask ASA for a statement of how the GAISE report was influential in defining statistics education at different levels.

AIM-Style Workshop: This workshop has two goals: to create a modeling course appropriate for high schools, and to explore the broader issues surrounding culture change needed to support such a course. For most high schools, it may be easier to embed modeling into other courses than to introduce a course entirely focused on modeling. However, such a course could be aspirational, in the way that statistics (particularly AP Statistics) has been aspirational for many schools. Other issues are likely to arise in preparation for the workshop or during this workshop collaboration:

- Does the International Baccalaureate curriculum provide any good models or approaches to developing and teaching such a class?
- What modeling courses at this level already exist and are they successful? (Preliminary investigations should bring existing information about curriculum materials or partnerships.)
- What content should go into such a course? Can it be connected to the wealth of existing resources (for example, ASA's Census at School, or the Citizen Science or KidNet programs)?
- How can we develop a pilot program of classes at select schools that have the capacity to support such a course?
- What kind of funding is needed to launch a pilot project?
- Should such a course serve as a capstone course in some way?
- How might such a program foster subsequent infusion of modeling throughout the high school and middle school mathematics and science curriculum?
- Can we develop a cadre of teachers with modeling expertise, possibly through a summer institute or an online course/leadership program? (Again, there is a large, existing knowledge base on this topic.)



- Need timeline for development
- Need a plan to disseminate and get new schools to adopt the class
- Would mathematical modeling circles (like math circles) be a good approach to launching such a model course around the country? What lessons can be learned from existing programs the Moody's Mega Math Challenge and its database of teacher-coaches?
- Evaluation of students and curricular materials during and after piloting process
Standardized tests? Attitude change?

Right now, the College Board is the source of new and revised AP courses. A few decades ago, the Woodrow Wilson Summer Math Institutes (1984-1993) ran programs on statistics, and – having attended – one had the understanding that you were responsible to do something with the experiences. Similarly, this working group could plan or organize a modeling course (a course on thinking) that is supported and validated in some regularized way by the “big four” mathematical sciences societies.

An AIM workshop could have three main outcomes: developing a modeling course that can win the mathematical sciences societies' stamp of approval; a plan to build a community of practice in modeling at the high school level (perhaps by a the Wilson model of an intensive period of work plus a follow-up activities); and a suite of high school research groups in mathematical modeling that parallels the math circles structure. We note that AIM proposals are due in October and require three or four tangible goals to be accomplished.

Infusion of modeling and models across the mathematics curriculum and connecting to other disciplines: A working group focused on infusion of modeling into courses across the curriculum should study how to directly support teachers and schools in launching modeling tasks within the existing curricular framework. For example, some standard math class questions can be adapted to be more reflective of a modeling approach. The recommendations should start with small steps – straightforward and specific goals to help teachers start incorporating models then modeling into



their courses. Small adaptations of existing practices (such as emphasizing describing slope with units always attached) could have a great effect on student understanding of mathematical modeling.

It is important not to overwhelm teachers who are already fully loaded with implementing the common core standards and managing an increase in standardized testing. The infusion working group should also consider how to motivate schools and teachers to use these adaptations, at least by making connections to the critical thinking and modeling practices of the CCSSM and the performance on PARCC or other standardized assessment tests. One possible “carrot” is that well-designed experience in mathematical modeling promotes critical thinking, thus students are likely to perform better on tests and other kinds of analytical tasks. Then connections to college success could be emphasized: when students come into college having mastered mathematical modeling, they'll provide a great resource for colleges to enrich their own curricular offerings. Professional societies' support could get colleges to want students who have taken this critical thinking class. Development of these recommendations must be in partnership with practicing teachers: teachers can self-identify for how they want to bring some of these practices into their classrooms, and the results of this working group should support teacher professionalism and autonomy.

Professional Development Working Group: Teachers have a lot to do, and any request to teachers to transform their practice must come along with opportunities for ongoing, effective professional development focused on the content and practices for mathematical modeling across the curriculum. Effective, respectful professional development is a critical element to the success of any curricular change. This working group should develop a range of PD programs or recommended programs that can support a wide variety of teachers. Issues that this working group will likely encounter:

- How to frame a research-based need for PD and/or online instruction in modeling?
- What should the content of such PD be?
- What variety of time commitments or structures would be most useful to offer?
- Should PD be offered for college credit? Or continuing education credits? At what institution(s)? Could we advocate for a salary differential based on this further education? We should examine the NCTM online modules – do they incorporate modeling in an explicit and effective way?
- In what non-traditional ways might we support teacher growth, e.g., teaching channel video clips?
- Should these PD opportunities be free or tuition based? And what should be the long-term commitment of participants?
- Ideally, pre-service secondary teachers should experience at least one well-done modeling course; but can our professional development make up for the widespread lack of such courses at the university/teacher college level?

The need for effective, reliable professional development is immediate – how do we act fast to create a supportive and high quality suite of opportunities for teachers? Can we partner with existing programs such as the Park City Mathematics Institute? One proposal from our group is to have a summer intensive institute of four weeks' duration coupled with extensive academic year follow-up: the idea is to build a community around modeling, in which people are encouraged and expected to do something with their new expertise.

Assessment: We see two approaches to assessment: (1) inspired by the GAISE report which includes a useful set of examples; and (2) assembling a library of existing assessments that have been peer-reviewed and endorsed in some appropriate way. A working group on assessment should develop a wide range of assessment questions and be open to more creative or open questions such as project-based assessments. With each question, we should include a rubric that helps distinguish good/fair/poor performance and conveys the purpose of the assessment. Again, assessments must make connections to the mathematical practices in the common core.

Inspired by the thoughtful Force Concepts Inventory from physics, we suggest developing a parallel evaluation for mathematical modeling, a modeling concepts and practices inventory.

Another successful assessment comes from the AP Statistics Exam: we suggest examining its last question, which poses a context students may not have seen and which is typically more open-ended. We suggest involving the illustrativemathematics.org and other communities of mathematical scientists and educators.

The deep question is how do you get to the heart of assessing the higher level (creativity, critical thinking) modeling education practices? A working group in assessment will have to involve members who are expert evaluators and expert teachers.

Curated Repository: We recommend a small group develop a proposal for a curated repository of modeling resources, preferably peer-reviewed. There is great need for an easy to use, centralized resource that is regularly updated using a solid review process. Such a repository should include research articles from the mathematical sciences, research articles from modeling education, and peer-tested resources. One central question is what form should such a repository take, what distinctions and categories should be made, standardized or systematic tagging schemes, and such. Such a project will also need a “reference librarian” or staff members who keep it updated and manage the review process. This requires sustained funding.

Looking at existing examples is critical; this is a huge project and we may not be the best group to make a proposal in this area. If the working group decides others are better suited to developing a new repository (or expanding and better publicizing an existing repository), their report should make recommendations about how to advance such a project. At minimum, we hope to see the following outcomes:

- A summary of best practices in modeling education and examples of these
- An overview of existing resources
- Identification of gaps and challenges in accessing existing resources
- Recommendations for collaborators, for an institutional home, and for possible funding sources.

We strongly encourage tapping the resources of all the mathematical sciences societies in order to take advantage of their substantial past work in developing education resources in mathematical modeling.

Public Awareness: We recommend that the professional societies take the lead in declaring 2015=Year of Mathematical Modeling, with activities inspired by MPE, Statistics2013.org and the Math Awareness Month project. A group of us should write the vision statement for professional societies to endorse. We can point to existing resources such as Math Moments and Math Matters, and we can attach low level and high level modeling questions to create a second page of activities for each flyer. We should partner with as many of the other public awareness projects as possible, starting with SIAM outreach such as the WhyDoMath project and the SIAMBlog.

The societies should promote or expand their promotion of

- modeling competitions K-16
- peer mentorship programs that connect colleges with high schools via student mentors
- a modeling day during Math Awareness Month to encourage sharing among high schools – perhaps there should be contests and presentations?
- connections with science centers and museums

This kind of work requires a huge inter-society effort to generate a big impact. A statement from ASA about their successful experience in defining statistics education at all levels might be enormously helpful in bringing other mathematical sciences communities together behind this effort.

Conclusion

The members of this large, high school focused working group came up with many ideas in each area of our recommendations, and at every stage we were strongly aware that these activities must be developed in concert with the Early Grades and the Undergraduate working groups. We viewed our charge as looking at large scale, systemic change. Encouraging teachers to go to a science fair is nice; however, empowering teachers to transform their practice to a more modeling-based approach is much more in line with what we want to see. We believe that the Modeling Across the Curriculum project is a worthwhile effort that has the potential to improve mathematics education in K-16 and that can enliven and enrich students and teachers in understanding the power and elegance of mathematical thinking. It is critical to have the major national societies deeply involved in this project and pushing their membership communities to support and assist in infusing modeling across the curriculum, establishing modeling courses, and in encouraging formal education K-16 to value modeling.

Drafts of a proposal to SIAM and the other societies for a GAIMME report and a discussion of the goals of an AIM-style workshop on mathematical modeling course development are included in Appendix E.

Undergraduate Working Group

Jeff Humpherys

The conference began with a charge from our opening speaker, Joan Ferrini-Mundy of the National Science Foundation, to think about effective ways to educate students at the crossroads of modeling, data science, information science, computational science, and computational thinking. After much discussion, our group identified two main pathways to help meet this goal. First, we identified the need for two different studies or reports that we thought should be commissioned to inform and educate the various stakeholders on the central role that mathematical modeling plays in society. Secondly, we suggested a greater role that SIAM and other professional organizations can and should play to help create and support communities of practitioners in applied mathematics education.

Recommended Studies and Reports

Broadly speaking we identified two primary challenges where we felt that studies or reports would be helpful and influential. The first challenge is to illustrate how mathematics connects to the rest of the world by identifying its past and current successes and articulating to STEM practitioners, and the public as a whole, the essential and centralizing role that mathematical modeling plays in innovation. The second challenge is to identify and disseminate more targeted strategies for mathematicians to attract and retain students into STEM fields through mathematical modeling.

Connecting Math to Reality

The first challenge or opportunity is about connecting mathematical modeling to the rest of the world. By looking at the world through the lens of mathematics, we see its majesty and ubiquity of mathematical modeling percolating through nearly all aspects of 21st Century discovery and innovation. Engineers use modeling and simulation to test designs, pharmaceutical companies model drug responses and carry out adaptive clinical trials to minimize the costs and potential harm associated with testing drugs on human subjects, and markets use mathematical models to buy and sell products and services in nearly every major industry from Wall Street to Main Street.

As a working title, we suggested a report called *Connecting Math to Reality*, which would explore the impact that modeling has had on the world, highlighting a number of modeling achievements in history that have greatly benefited society and the world as a whole. To support this report, a series of vignettes would provide a diverse set of examples to help attract a broad readership and to provide practitioners and educators with examples to draw from as they communicate with the public about the mathematical sciences. Along these lines, it would be good to help educate students, teachers, guidance counselors, and parents on what modeling is and why mathematical modeling is important. In addition, we also recommended the development of some educational modules that could go with the report that could be used in classrooms. This

would be particularly useful to guide the discussion on modeling given the wide adoption Common Core State Standards.

Another desired outcome of this report is to follow Ferrini-Mundy's challenge and provide the scientific community with guidance on how to educate students, largely in higher education, at the crossroads of modeling, data science, information science, computational science, and computational thinking. These disciplines are moving quickly and there are several communities, departments, and research groups that are intersecting and yet not really communicating with each other. There are concerns that artificial disciplinary silos might form and that this could be bad for science, in particular it would be bad for the students who make up the next generation of scientists. Along these lines there are concerns that these silos will use different jargon for the same ideas thus creating the need for translation in order to do interdisciplinary work. It is better to use a common language to the extent possible, and that common language should be mathematics.

In addition to the inefficiencies that can arise from this lack of cross-fertilization, it seemed desirable to stimulate cooperation, at least at the educational level, to avoid different departments teaching the same or highly similar content. With the proliferation of ideas, there's pressure to create new departments within universities, thus taxing the administrative overhead and making institutions fractured and top heavy.

It has been suggested that applied and computational mathematics has an opportunity to be the glue that connects these fields together, to help facilitate cross-fertilization, but in order to do so, substantial curricular and cultural changes will be necessary.

Modeling and the Pipeline

It is dangerous to learn to fly while flying---mistakes come at a very high cost! To avoid this, we have flight simulators that allow pilots and trainees to make mistakes virtually and learn from them without having to experience the tragedies that come from real mistakes. Similarly, it is dangerous to test out new products, services, designs, and policies in a real-world setting, such as a business or government agency, without first testing ideas in the virtual world.

Of course a major difference between a flight simulator and a market simulator is that the flight simulator has laws of physics that govern the dynamics of the airplane and allow the simulator to be nearly perfect in its representation of actual flight. In business or government, however, natural laws are replaced by market responses from both consumers and competitors, and so models tend to be complex, incremental, and uncertain instead of absolute and largely well understood. Indeed there is often a large gap between theory and practice when human interaction is concerned, and in many cases there isn't even a reasonable theory. Nonetheless, the idea is the same. Virtual experimentation is replacing many aspects of real-world implementation and the demand for modelers is rapidly increasing.

This demand translates into jobs, and so our second recommended report or study is to find ways to attract and retain students into STEM fields through mathematical modeling—we need to study and understand the STEM pipeline and the role that modeling plays, or can play, to stimulate growth and vibrancy in the quantitative disciplines. With the projected future shortfall of STEM graduates as

described in Engage to Excel and the call for a 34% increase in STEM majors, we see an opportunity to strengthen the pipeline through mathematical modeling, and that opportunity needs to be studied and reported to the community.

The information age has provided us with both massive amount of data and substantial computational resources whereby we can extract useful information. The computational and data sciences are a hot area and companies are clamoring to find people who can innovate in a data-rich environment. There are substantial opportunities for the mathematical community to attract and retain students if we can adapt to this growing opportunity.

One question that was raised in the workshop was whether there are other entrances into the mathematical sciences that follow an alternative track different than the usual calculus approach. Could a freshman math modeling class bring students into applied mathematics who might not otherwise be majors? Moreover, with the calculus track, is there a new approach that would improve educational outcomes? These questions should be addressed in this study.

Recommendations for Professional Organizations

In addition to the two reports or studies described above, our group identified opportunities for professional organizations to create and support communities of practitioners in applied mathematics education.

SIAG on Applied Math Education

One of the group's recommendations was that SIAM create an activity group on Applied Math Education. This would provide numerous opportunities for cooperation, collaboration, and recognition. Examples include conferences, sessions at the annual meeting, email lists, SIAM-backed blogging, and even perhaps an online magazine. There could also be awards given to departments and individuals recognizing their contributions.

Another benefit of an activity group would be the ability to pull people together to serve the community in a coherent and cohesive way. There was great interest in our group in having a curated library of trusted resources, with ratings, moderation, reviews, and ample metadata, e.g., synopsis, categorize by area, pedagogical type, review of literature, and reviewed resources, so that people can find reliable resources to use in their classrooms and even participate in the development efforts if desired. This would open substantial opportunities for both collaboration and dissemination. There are also great opportunities for social networking, tweeting, blogging, etc., to further stimulate collaboration and cooperation, and volunteers within the activity group might make good moderators and reviewers for such content.

There was some discussion on how to differentiate the activity group from SIAM's education committee. The underlying maxims guiding who does what seemed to fall on the activity group existing to support research and education activities surrounding efforts in academia, whereas the Education Committee will serve the SIAM



community as a whole. For example, SIURO will be managed by the SIAM Education Committee, but a conference on Applied Math Education would be run by the activity group.

SIAM Education Committee Opportunities

Our group also identified a need for cooperation across professional organizations and it was recommended that the SIAM Education Committee continue to make and establish connections with the education VPs of other societies and organizations. Examples of organizations include MAA, AMS, AAAS, NCTM, ASA, AERA, AMTE, SIAM, IEEE CSS, ACM, COMAP, SCB, SMB, INFORMS, RUME, AMTYC, APS, CBMS, MSO, AMTA, CSEE, to name a few. It would also be helpful to connect to centers that are modeling-friendly such as DIMACS, Cause, etc.

By connecting with these organizations, there's an opportunity to address a number of important questions and try to get some consensus around some of the larger educational issues in the broader STEM community. For example, what are the best practices in accessing and evaluating educational programs dealing with modeling? How does one judge creativity and higher-order thinking, how does one judge the quality of a program, the learning outcomes, program outcomes, etc.? What are the goals of a good modeling program? What does it mean to be college ready? What are the best ways to remediate? How can these groups work together to achieve better outcomes?

Key Discussion Points

The following are some discussion points that emerged from our meeting. While we did not want to make any specific recommendations in these areas, we felt that it would be worth considering the problems in our discipline and the trends that can be observed in academia and industry.

- The mathematics community is largely unaware of how math is used in other quantitative disciplines. The math curriculum

has not changed much since the 1960s, and yet other related disciplines have changed substantially, and so we are really out of touch (speaking broadly not individually).

- As an example, the singular-value decomposition (SVD) in linear algebra is a widely used technique in statistics, computer science, engineering, finance, and economics, and yet many pure mathematicians are unfamiliar with the topic, in large part because good numerical algorithms weren't developed until the 1960's and 1970s. To many mathematicians, linear algebra is the study of the algebraic properties of vector spaces and linear transformations. Some mathematicians pay little attention to the geometric and operator-theoretic properties of the field where applications are most prevalent. As so many other disciplines use the SVD, it is not only important that mathematicians understand what it is, but also teach it thoroughly in linear algebra and matrix analysis courses.
- As another example, the mathematics community is largely unaware of what Bayesian Statistics is and the role that it plays in emerging fields such as machine learning and natural language processing. Latent Dirichlet Allocation (LDA) was specifically mentioned as a new idea in applied Bayesian statistics. This and other related techniques are generative modeling methods that are quite powerful.
- Compressed sensing was also mentioned as a new hot area in computational harmonic analysis. How should it and other new and emerging areas of applied mathematics be woven into the curriculum so that students learn these methods and can apply them when they get into the workforce? Even at the undergraduate level, compressed sensing could be introduced alongside l1-regularization so that problems where sparse solutions are wanted can be obtained.
- The traditional undergraduate degree in mathematics does not prepare students for careers in industry. There are very few topics, if any, that are traditionally covered in mathematics that were developed after 1900. As a result, graduates in mathematics have few qualifications and little preparation in the workforce unless they seek (usually on their own) a background in computer programming or statistical modeling. Without these skills, math majors struggle to get the high-paying jobs that related STEM graduates get.
- It's time to grow up: There was discussion surrounding the idea that our way of life (for our discipline) can't persist if we continue to fail to connect to other disciplines and provide students with the mathematics that they need to succeed in the workforce. Over time, if we don't change, we will have resources redirected away from us. One of the participants said, "It's time to grow up".
- What algorithms should be taught in the undergraduate curriculum? In a traditional curriculum, in Calculus, Newton's and Simpson's Rule are usually taught for one-dimensional problems and Euclid's division algorithm is taught in abstract algebra. Many will struggle to come up with examples beyond that. Below are families of algorithms that are accessible to undergraduates and that could be considered at some level in a modern curriculum:
 - Encryption algorithms: finding pseudo primes with Fermat's little theorem, RSA, Diffie-Hellman key exchange
 - Solving linear systems: Jacobi, Gauss Seidel, Successive Over-Relaxation (SOR), Krylov methods such as GMRES
 - Signal processing and time-series analysis algorithms: DFT, FFT, ARMA, ARIMA
 - Compression algorithms: Huffman encoding, wavelets, LZW
 - Tree search algorithms: AVL trees, BW-trees, B-trees
 - Constrained optimization: simplex algorithm, interior-point methods
 - Unconstrained optimization: Newton's method, conjugate-gradient, quasi-Newton methods
 - Markov-Chain Monte Carlo Methods: Gibbs Sampling, Metropolis-Hastings, Metropolis
 - Matrix Decompositions: SVD, QR, LU
 - Graph Algorithms: MST, TSP, BFS, DFS, greedy algorithms
 - Markov Decision Processes: multi-armed bandit problems
 - State Estimation: recursive least squares, Kalman filtering, particle filters
 - ODE Solvers: Runge-Kutta, boundary-value solvers
 - PDE Solvers: Finite-element and finite-difference methods
- Topics that were discussed and recommended that could/should be included into the curriculum are:
 - Design, analysis, and optimization of algorithms
 - Probability and stochastic processes
 - Bayesian statistics, machine learning, and data analytics
 - Dynamical systems and Control Theory
 - Optimization
- Technical Skills: Students should know how to work with data. Web scraping, regular expressions, relational databases. Additionally students who are good with scientific visualization, low-level programming (C/C++), high-level scripting (Python, R), and distributed computing technologies (such as MPI, Hadoop/Map-Reduce) will have a substantial advantage in the workforce.



Conclusions and Recommendations

The report on MaC I stated that “The most obvious conclusion to draw from the foregoing is perhaps that this short workshop could do little more than scratch the surface ...” in the development, recruitment, retention and education of a strong pool of STEM undergraduate majors. One of the key recommendations was that:

“There should be a follow-up workshop of longer duration which can explore questions raised in this report, the linkages between the different themes, and reach greater specificity on research questions.

- A minimum of 2.5 days seems appropriate, with
- Expanded participation including many of the attendees from the first workshop, and including
- Both pairwise and three-way interactions among themes to explore connections.

An important outcome of the second workshop will be to identify small leadership teams for each theme. The workshop steering committee would begin that process in the planning stage.”

The foregoing demonstrates that this objective was met. Substantial progress was made in each of the three groups resulting in specific recommendations and action items. One overarching recommendation was again that there should be a third such workshop to address some of these specifics, and to disseminate progress that has been made in the interim.

A key recommendation from the undergraduate group, and endorsed by each of the others bears repetition here:

Recommendation 1

One of the group’s recommendations was that SIAM create an activity group on Applied Math Education. This would provide numerous opportunities for cooperation, collaboration, and recognition. Examples include conferences, sessions at the annual meeting, email lists, SIAM-backed blogging, and even perhaps an online magazine. There could also be awards given to departments and individuals recognizing their contributions.

In fact this recommendation has already been implemented. SIAM Board and Council approved the establishment of SIAG/ED at their meetings in July 2014 and the group begins operation officially on January 1, 2015. It is hoped that the first conference of the SIAG can coincide with MaC III and therefore bring more people into the effort and facilitate both the dissemination and further development mentioned above.

Several specific suggestions from the different groups are included in the Executive Summary at the beginning of the report.

Each of the thematic discussions was very fruitful. Important topics were identified as recommendations or action items by each group.

The early grades recommendations centered around the need for professional development and pre-service training for teachers who have typically little awareness of mathematical modeling. The group developed a lengthy list of Action Items which can perhaps be summarized as

Recommendation 2

Develop strong professional development and teacher training programs, materials and support networks to provide experience, understanding and skills in mathematical modeling at levels appropriate for use in early grades classrooms.

This is a major undertaking. It probably requires the creation of some specialist teachers even for the early grades. Materials that would be needed include:

- Producing materials, including classroom posters, videos, materials for teacher training and professional development, released standardized test items and classroom projects that help communicate what mathematical modeling is.
- Developing professional development programs that train teachers (and perhaps also math specialists, district leaders, mathematicians and parents) how to do mathematical modeling and facilitate mathematical modeling for early grades.
- Creating peer-networks and social networking sites for teachers to share ideas, and locate materials that have been class tested. This site should include “promotional” videos perhaps of discussions with, or interviews of, experts and teachers to explain the modeling process and its importance.

The third Modeling across the Curriculum workshop and SIAG/ED conference should help with disseminating progress to date and advancing these goals. This recommendation clearly also necessitates involvement of teacher educators, supervisors of mathematics and mathematics education expertise. That would represent a significant broadening of the MaC initiative.

The High School working group made several recommendations which can be summarized in terms similar to the Early Grades’ Recommendation 2 above:

Recommendation 3

Develop strong professional development programs, curricular and assessment materials, and develop working groups to investigate different strategies for introducing modeling into the high school. Some specifics components are

- produce Guidelines for Assessment and Instruction in Mathematical Modeling Education (GAIMME) along the lines of the ASA’s GAISE Report.
- propose and run an American Institute of Mathematics Workshop focused on developing a high school mathematical modeling course and standards for secondary modeling education.
- create Working Groups to study different strategies such as Infusion of Modeling into high school curricula, Professional Development for teachers to improve or develop their expertise, and Assessment
- develop a curated Repository of peer-reviewed and tested materials covering projects, curricular components, career and public awareness

As in the first proposal, there is already progress to report. SIAM and the Consortium for Mathematics and its Applications, COMAP, have agreed to fund a workshop specifically charged with developing a GAIMME report.

Recommendation 1 above originated with the undergraduate curriculum group and was quickly endorsed by the other two groups. The Undergraduate section of the report also calls for two major reports which would be valuable throughout the educational continuum.

Recommendation 4

Two major reports similar to those produced for the National Academies should be commissioned:

- Connecting Mathematics to Reality, and
- Modeling and the Pipeline

The first will have value to educators, students and advisors at all levels. In particular it will arm teachers with answers to the “Why do

I need to learn this?” or “When will I ever use this?” questions. Note that the way in which the second of these is worded almost pleads for an applications and modeling perspective to mathematical education. The second proposed report speaks to the vital role mathematical sciences play in the development of an appropriately prepared and skilled workforce.

It is plain from the report that much work still needs to be done. Much of this work can continue among the various teams and communities that have developed. There will be a need for a periodic reconvening of a more general group and so a third Modeling across the Curriculum workshop should be planned. Combining it with the first biennial conference of the new SIAM Activity Group in Applied Mathematics Education will enable the continued dialogue among the interested groups and the broadened participation that the conference would facilitate.

References and Further Reading

- [1] American Statistical Association, *Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report*, American Statistical Association, Alexandria VA, 2005
- [2] Ang, K.C. (2001). Teaching Mathematical Modelling in Singapore Schools, *The Mathematics Educator*, 6(1), 62-74.
- [3] Kelly Black and Dawn Meredith. *Studio calc/physics: Combining calculus, physics, and active learning*. In 123rd National Meeting of the AAPT, Rochester NY, July 2001.
- [4] Kelly Black and John Geddes. *Preparing undergraduate students for research*. In SIAM Mini-symposium on Undergraduate Programs and Research Projects in Applied and Computational Mathematics, San Diego, CA, January 2002. Joint Mathematics Meetings.
- [5] K.M. Bliss, K.R. Fowler & B.J. Galluzzo, *Math Modeling: Getting Started and Getting Solutions*, SIAM, Philadelphia, 2014
- [6] D. Bransford, A. L. Brown, and R. R. Cocking, *How People Learn: Brain, Mind, Experience and School*. National Academies Press, Washington, DC, 2000.
- [7] C. Brewer, and D. Smith (eds.), *Vision and Change in Undergraduate Education: A Call to Action*. AAAS, Washington, DC, 2011
- [8] Carpenter, T. P., Ansell, E., Franke, M. L., Fennema, E. & Weisbeck, L. (1993). Models of problem solving: A study of kindergarten children's problem-solving processes. *Journal for Research in Mathematics Education*, 24(5), 427-440.
- [9] Carpenter, T., Fennema, E., Franke, M., L. Levi, and S. Empson. *Children's Mathematics: Cognitively Guided Instruction*. Portsmouth, NH: Heinemann, 1999.
- [10] Cirillo, Michelle. "What are some strategies for facilitating productive classroom discussions?" Research Brief, NCTM, posted January 13, 2013. Retrieved from: http://www.nctm.org/uploadedFiles/Research_News_and_Advocacy/Research/Clips_and_Briefs/research%20brief%2020%20-%20strategies%20of%20discussion.pdf
- [11] Common Core State Standards Initiative, Common Core State Standards: Mathematics, <http://www.corestandards.org/the-standards/mathematics> & <http://www.corestandards.org/Math/Practice>
- [12] Engage NY. "Mathematical Modeling: What it is; What it looks like in the classroom; Why it is so important" [PowerPoint] February 2013 "http://www.engageny.org/resource/network-team-institute-materials-february-4-7-2013-mathematics-6-12-pd-day-two, Facilitators: Eric Robinson and Teri Calabrese-Gray
- [13] Greer, B. (1997). Modelling Reality in Mathematics Classrooms: The Case of Word Problems, *Learning and Instruction*, 7(4), 293-307.
- [14] L. J. Gross, 2004. *Interdisciplinarity and the undergraduate biology curriculum: finding a balance*. *Cell Biology Education* 3:85-87.
- [15] L. J. Gross, 2005. *It's Wednesday: take two*. Pp. 119-121 in *The Art of College Teaching: Twenty-eight Takes*. M. Kallet and A. Morgan (eds.). University of Tennessee Press, Knoxville.
- [16] L. J. Gross, 2012. *Some lessons from fifteen years of educational initiatives at the interface between mathematics and biology: the entry-level course*. To appear in: *Undergraduate Mathematics for the Life Sciences: Models, Processes and Directions*.
- [17] Herbel-Eisenmann, B., Steele, M. D., & Cirillo, M. (in press) (Developing) Teacher Discourse Moves: A framework for professional development. *Mathematics Teacher Educator*.
- [18] Kinch, D. (Dec 2011). LACOE. [L.A. County Office of Education] commoncore.lacoe.edu/resources/training.../3_modeling_20111209.ppt
- [19] Glenn Ledder, Jenna P. Carpenter and Timothy D. Comar (editors) 2012 (to appear) *Undergraduate Mathematics for the Life Sciences: Models, Processes, and Directions*. Mathematical Association of America Notes.
- [20] Richard Levins,. 1968. *Evolution in Changing Environments*. Princeton University Press.
- [21] Mathematics Assessment Project, Math Assessment Resource Service, University of Nottingham (2012) <http://map.mathshell.org/materials/lessons.php?taskid=422&subpage=problem>
- [22] Meyer, Bryan. Tasks, teacher expectations, and what we value. 10 Oct 2012 (blog): <http://www.doingmathematics.com/2/post/2012/10/tasks-teacher-expectations-and-what-we-value.html>
- [23] National Research Council, *The Mathematical Sciences in 2025*, National Academies Press, Washington DC, 2013
- [24] Pollak, H. O. (2012). Introduction – What is Mathematical Modeling?. In Gould, H., Murray, D. R., & Sanfratello, A (Eds.), *Mathematical Modeling Handbook* (pp. viii-xi), Bedford, MA: The Consortium for Mathematics and Its Applications. Available online at: www.comap.com.
- [25] President's Council of Advisors on Science and Technology, PCAST, *Engage to Excel: Producing One Million Additional College Graduates with Degrees in Science, Technology, Engineering, and Mathematics*, Office of Science and Technology Policy, Washington DC, February 2012, http://www.whitehouse.gov/sites/default/files/microsites/ostp/pcast-engage-to-excel-final_2-25-12.pdf

- [26] Patrick D. Schalk, David P. Wick, Peter R. Turner, Michael W. Ramsdell IMPACT: Integrated Mathematics and Physics Assessment for College Transition Frontiers in Education, San Antonio, Texas, October 2009
- [27] P. D. Schalk, D. P. Wick, P. R. Turner, and M. W. Ramsdell, Predictive assessment of student performance for early strategic guidance. Rapid City, SD, October 2011, ASEE/IEEE Proceedings of the 41st annual FIE (Frontiers in Education) Conference.
- [28] Schuster, Lainie. Good questions for math teaching, Grades 5-8. Math Solutions Publications, Sausalito., SERP Institute, 2014. Diagnostic Teaching in Mathematics: Poster Problems. Retrieved from: http://math.serpmedia.org/diagnostic_teaching/team/
- [29] SIAM Modeling across the Curriculum: Report on a SIAM-NSF Workshop, Society for Industrial and Applied Mathematics, SIAM, Philadelphia, 2012
- [30] SIAM Working Group on CSE Education (Linda Petzold, Chair) Graduate Education in Computational Science and Engineering, SIAM Review 43 (2001) 163-177
- [31] SIAM Working Group on Undergraduate CSE Education (Peter Turner, Linda Petzold, Co-Chairs) Undergraduate Computational Science and Engineering Education, SIAM Review 53 (2011) 561-574
- [32] SIAM, Moodys Mega Math Challenge, <http://m3challenge.siam.org/>
- [33] Smith, Margaret S., and Mary Kay Stein. "Selecting and Creating Mathematical Tasks: From Research to Practice." Mathematics Teaching in the Middle School 3 (February 1998): 344-50.
- [34] Smith, Margaret S., and Mary Kay Stein. Five Practices for Orchestrating Productive Mathematics Discussions. NCTM, Reston, VA, 2011.
- [35] Stocker, David. Math that Matters. CCPC, Canada 2007.
- [36] Suh, Jennifer M., et al. Being an environmentally friendly package engineer Teaching Children Mathematics 20.4 (2013): 261-263.
- [37] Suh, Jennifer, and Padmanabhan Seshaiyer. Mathematical Practices That Promote 21st Century Skills, Mathematics Teaching in the Middle School 19.3 (2013): 132-137.
- [38] Sullivan, Peter and Pat Lilburn. Good questions for math teaching: Why ask them and what to ask, K-6. Math Solutions Publications, Sausalito, 2002.
- [39] P.R.Turner, A Predictor-Corrector with Refinement for First-Year Calculus Transition Support, PRIMUS 18 (2008) 370-393
- [40] P.R.Turner, Teaching scientific computing through projects, J of Eng. Ed. 90 (2001) 79-83
- [41] P.R.Turner & K.R. Fowler, A holistic approach to Applied Mathematics Education for Middle and High Schools, Education Interface between Mathematics and Industry, EIMI 2010, Proceedings, Lisbon, Portugal, pp 521-531
- [42] The Mathematical Education of Teachers http://www.cbmsweb.org/MET_Document/
- [43] The Consortium for Mathematics and Applications, COMAP, <http://www.comap.com/undergraduate/contests/mcm/>
- [44] Van Dooren, W., De Bock, D., Vleugels, K., & Verschaffel, L. (2011). Word Problem Classification: A Promising Modelling Task at the Elementary Level. In Kaiser et al. Trends in Teaching and Learning of Mathematical Modelling (pp. 47-55), New York: Springer.
- [45] Vanderbilt University Technology Group, The Jasper project: lessons in curriculum, instruction, assessment, and professional development. Lawrence Erlbaum, 1997.
- [46] Verschaffel, L., van Dooren, W., Greer, B., & Mukhopadhyay, S. (2010) Reconceptualising word problems as exercises in mathematical modelling. Journal für Mathematikdidaktik, 1, 9–29.
- [47] Wolfe, Patricia. (February 1987) "What the 'seven-step lesson plan' isn't!" Educational Leadership. p.70-71.
- [48] <http://www.smarterbalanced.org/smarter-balanced-assessments/>
- [49] [<http://www.parcconline.org/parcc-assessment>
- [50] <http://www2.ed.gov/programs/mathsci/index.html>
- [51] http://www.rethinkingschools.org/archive/27_03/27_03_gutstein.shtml

Appendix A

Workshop Agenda

Second SIAM-NSF Workshop on Modeling across the Curriculum January 12-14, 2014, Alexandria VA

Themes

Undergraduate Curricula	Coordinator Jeff Humpherys
Middle and High School STEM/ Modeling Courses	Coordinator Katherine Socha
Early Grades	Coordinator Rachel Levy

Topical areas within each theme

Programs:	Courses, Programs, Degrees, Summer Experiences
Materials:	Books, Videos, Software, Posters, Websites
Training:	Pre- and In-service, Ways of interacting with teachers, faculty, TAs and students

Sunday January 12

1:00	Arrival and Registration
1:30	Welcome and overview of the meeting Joan Ferrini-Mundy, Assistant Director for Education and Human Resources, NSF
	Peter Turner Introduction to the meeting and report on MaC I
2:30 – 4:00	Introductory panel on the themes Rachel Levy Bringing modeling to the early grades Katherine Socha HS modeling course and curriculum development Jeff Humpherys Undergraduate curriculum: Applied and Computational Math
4:00 – 4:30	Math Models: Getting Started and Getting Solutions Moodys Problem Writing Committee Report (Karen Bliss, Katie Fowler, Ben Galluzzo)
4:45 – 6:00	Keynote Address Mark Green Math2025
6:30 – 8:30	Reception and networking discussions in the Sheraton provided by SIAM

Monday January 13

7:30 – 8:00	Breakfast provided at ASA (Second Floor conference room)
8:00 – 8:30	Intro Slides on each Theme Area Three “theme rooms” so we can have non-competing presentations
8:30 – 10:00	Working group sessions, Moderators, and Recorders U: Undergraduate curricula Jeff Humpherys Bill Kolata H: HS STEM Curriculum Development Katherine Socha Michelle Montgomery E: Early Grades Rachel Levy
10:00 – 10:30	Break
10:30 – 12:00	Mixed groups (approx. 1/3 of each of the main theme groups in each mixed group, and try to rotate those in subsequent mixed groupings) U1,H1,E1 Undergraduate curricula Jim Crowley Bill Kolata U2,H2,E2 HS STEM Curriculum Development Rebecca Nichols Michelle Montgomery U3,H3,E3 Early Grades Peter Turner
12:00 – 1:00	Lunch and informal discussions (Second floor conference room)

Appendix A

Workshop Agenda

1:00 – 3:30	Working groups sessions U: Undergraduate curricula H: HS STEM Curriculum Development E: Early Grades	Jeff Humpherys Katherine Socha Rachel Levy	Bill Kolata Michelle Montgomery
3:30 – 4:00	Break		
4:00 – 5:30	Mixed groups U2,H3,E1 Undergraduate curricula U3,H1,E2 HS STEM Curriculum Development U1,H2,E3 Early Grades	Jim Crowley Rebecca Nichols Peter Turner	Bill Kolata Michelle Montgomery

Revised Agenda for Tuesday January 14

7:30 – 8:00	Breakfast (Second floor conference room)
8:00 – 8:45	Plenary session (First floor conference room) Brief intro to the day followed by status reports from each them area
8:45 – 9:00	Break
9:00 – 12:00	Working group theme group sessions, Moderators, and Recorders Prepare final report out
12:00 – 1:00	Lunch and informal discussions (Second floor conference room)
1:00 – 2:30	Theme groups report out to whole group (First floor conference room)
2:30 – 4:00	Theme groups work on next steps and assigning roles to team members
4:00 – 4:30	Closing and steering committee discussion on report preparation

Appendix B

The Working Groups

The Principal Investigators and NSF participants were not assigned to particular working groups but were observers and occasional participants in all. The overriding theme of all groups was Modeling across the Curriculum considered in the three different thematic areas.

Early Grades

Moderator: Rachel Levy
Participants: Matthew Ellinger
Padhu Seshaiyer
Michelle Cirillo
Stacy A Brown
Laura Pahler
John Pelesko

High School

Moderator: Katherine Socha
Recorder: Michelle Montgomery
Participants: Ben Galluzzo
Dan Teague
Diana Fisher
Katie Fowler
Oana Pascu
Richard Sisley
Sharon Hessney
Sol Garfunkel
Andrew Cagliaris
Lauren Leischer

Undergraduate Curricula

Moderator: Jeff Humpherys
Recorder: Bill Kolata
Participants: Giampiero Campa
John David
Lizette Zietsmann
Lou Gross
Mark Green
Reza Malek-Madani
Joe Malkevitch
Kelly Black
Simon Taverner
Dennis Pearl
Robin Lock
Richard Alo
Ron Buckmire

Introductory Slides

Each participant was asked to prepare a single slide to highlight a vision or an example that could help seed the discussion. These were presented during the first Working Group session.

Early Grades

Modeling Across the Curriculum

Early Grades

WORKING WELL:

"Poster Problems" for Diagnostic Teaching in sixth and seventh grade mathematics.

- bring the thinking of students out into the open so teachers can use it to promote rich discourse
- allow for a range of solution approaches – all valid, but some using more basic math than can seamlessly lead to using more advanced math
- give teachers lots of information as teachers complete tasks so they see and hear how students are advancing

Here is a [new website](#) with 12 problems prepared specifically for this approach, along with lesson plans and work samples. All include modeling



ONGOING NEED:

Compelling professional development for first and second grade teachers in using modeling.

- From Grade 2 CCS: "Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem."
- Here are two teachers who are trying some great things, but they are still struggling to have students use the models meaningfully or attach them to the relevant situation or equation.

Link to teacher [Sam Lampkin's Website](#)
(used with permission)



Wiley, J. (1998). *Strategic Educational Research: The Research Agenda*.

Rachel Levy, Harvey Mudd College (Early Grades)

Industrial Mathematics Projects

- Successful at the college level (+ study groups)
- Students in K-6 are great at identifying problems and creative solutions.
- Could they work with their school or community businesses to

Critical Needs

- For common core state standards, work with companies to create released test items for standardized tests.
- Test items should encourage/inspire/require teachers to engage their student in meaningful and mathematically rich modeling activities.
- Communicate to students and teachers what mathematical modeling involves and what modelers do – create posters/classroom materials.
- Navigate the tension between (a) problem-based iterative modeling with multiple possible solutions and techniques and (b) models that motivate a pre-determined mathematical technique.

Bringing modeling to the early grades



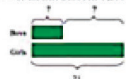
Modeling in Grades K-6

Laura Pahler
Pomona Unified School District, CA

Successes

Singapore Math Curriculum

- Use of number disks to represent place value
- Use of number bonds and bar
- models to demonstrate the
- relationships between



Needs

Authentic Math Problems

- Begin with a real-world, open-ended, unsolved problem
- Choose the level of accuracy and constraints desired
- Exercise creativity in choosing the model
- Integrate algebra, geometry, probability, and statistics



Michelle Cirillo, University of DE, Middle Grades

Critical Need: Mathematical Modeling Tasks

Tasks with these features:

- *Explicit* attention at the beginning of the process of getting from the problem outside of mathematics to its mathematical formulation.
- An explicit reconciliation between mathematics and the real-world situation at the end. (Pollak, p. 649)
- Requiring assumptions, creativity, decisions (CCSSM)

Adventures of Jasper Woodbury

- materials designed to bridge the gap between natural learning environments and school learning environments
- provide...a chance to see that school knowledge can be used to solve real problems.
- "embedded data design"
- solved by generating and solving the interconnected sub-problems in order to solve the overall problem.

The Jasper Project, The Cognition & Technology Group at Vanderbilt (1997)

Appendix C

Introductory Slides

High School

Modeling Across the Curriculum High School

Modeling Dynamic Feedback Systems



Materials Available:

Lessons in Mathematics: A Dynamic Approach (grades 8 – 14)
Modeling Dynamic Systems: Lessons for a First Course (grades 9 – 16)
(More at the Creative Learning Exchange: The Shape of Change, Dollars and Sense I & II... (K – 12); the Waters Foundation (K – 8); iSee systems, inc (9 +))

Training for teachers (online: Summer 2014):

Introduction to System Dynamics Modeling for Math and Science Instruction

Need: More materials (lessons), more instructors, intermediate online SD course

Level: Grades 7 – 12

Contact: Diana M. Fisher, Portland State University, System Science Dept.

diana.fisher@frontier.com or fischerd@psu.edu



www.cmodelingdynamics.com

STAT & PROB SIMULATIONS

Grades 7 – 12
Sharon Hessney
MA Mathematics & Science Initiative

SIMULATIONS in Advanced Placement Statistics

- Simulation of random behavior and probability distributions
- Simulation of sampling distributions
 - Textbooks: 5 recommended text use simulations
 - Software: teaching (Fathom), analysis (R, JMP, StatKey), and applets
 - Training: Summer Institutes lead by certified, College Board leaders
 - Other: AP Stat Community discussion group

SIMULATIONS in Common Core State Standards

- Grade 7
 - Use simulated data to draw inferences about population (7.SP.2)
 - Design and use a simulation for compound events (7.SP.8c)
- Grades 9 – 12
 - Decide if model is consistent with simulated results (S-IC.2)
 - Use simulated data to estimate a population parameter (S-IC.4)
- Textbooks:
- Software:
- Training:

Four Courses from the W. M. Keck Curriculum Project

FEATURES:

MINIMIZING A SINGLE TOPIC

PEDAGOGY:

INQUIRY AND GUIDED INQUIRY—

ACTIVE STUDENT ENGAGEMENT REMOVED

THE CHALLENGE OF ENCOURAGING TEACHERS TO ADOPT
PART OR ALL OF THE CURRICULUM—AN OPPORTUNITY TO
USE NEW METHODS OF COMMUNICATION

Richard Sibley
Loudon County Academy of Science
Richard.Sibley@kps.org



Dan Teague
NC School of Science
and Mathematics
Grades 7–12

"As a teacher, I felt unprepared to coach the students. Some of the papers were difficult for me, and I didn't know really how to teach the students to develop models in such an open ended situation. I would like to see you guys do a workshop for coaches. I come from a theory based math background and want to coach a team, but feel inadequate when it comes to the modeling scenarios. I've even considered going back to take some applied math classes."

Modeling Across the Curriculum (page 4)

What Works?

Woodrow Wilson Summer Math Institutes (1984-93)
NCSSM Statistics Leadership Institute (1999)
MAA PREP Supporting High School Research in Mathematics (2011-12)

What is Needed?

Experience is the key. You must look at modeling from the inside-out; this is minds-on mathematics and students must be willing to get their minds dirty.

Follow-up is essential. Teachers need on-going support, a second-taught if possible.

Discover the model in you!

LEVEL: GRADES 7-12
CURRENT AFFILIATION: PEDDIE SCHOOL, HIGHTSTOWN, NJ

NAME: ANDREW V. CALDERON



1. Existing Program/Materials/Training Methodology:

(Based on research conducted through the auspices of a curriculum development grant last summer to develop a stand alone one term course in Mathematical Modeling to be offered as a spring term elective)

- The COGAP HLEAP content problems as well as the Moody's Mega Math (High School Mathematics Contest in Mathematical Modeling) provide excellent experiences for a small number of students with a fairly sophisticated understanding of the modeling process. There needs to be a similar commitment to developing sequences of curricular units designed to develop modeling concepts, skills, and experiences for a broader population and starting from the early years (K-6) and extending into the Middle/High school years.
- A practical approach is for teachers to identify appropriate problem settings already referenced in materials currently being adopted and revise the problems to be consistent with those characteristics reflected in authentic and open ended modeling investigations.
- A more radical approach is to use authentic modeling scenarios as the vehicle to motivate the development of concepts and skills outlined in the 7-12 Common Core Curriculum Standards (CCCS).

2. Critical Needs: (Based on my observations as a classroom teacher and 7-12 mathematics coordinator)

- Communication with, and education of our constituents (colleagues, administrators, parents, and students) regarding the Role of Mathematical Modeling in the Common Core Curriculum ("What really matters in Mathematics Education is learning and mastering the modeling process" H. Pollak, Teachers College, Columbia University)
- Development of Teaching Methodologies grounded in both research and classroom practice designed to implement the integration of Mathematical Modeling concepts and skills throughout the 7-12 CCCS
- Development of Teaching and Learning Materials designed to reflect the integration of Mathematical Modeling concepts and skills throughout the 7-12 CCCS
- Training of Teachers in grades 7-12 to implement the modeling concepts and skills as articulated in the 7-12 CCCS
- Development of Assessment Tools to evaluate the effectiveness of both teaching methods and learning materials in the instruction of mathematical modeling concepts and skills throughout the 7-12 CCCS

Modeling Across the Curriculum (SIAM-NSF Workshop)

Jan. 18th - 19th, 2014, Alexandria VA

Appendix C

Introductory Slides

Moody's Mega Math M³ Challenge



- Math modeling contest for high school students started in 2006.
- Motivation (\$ prizes) and no fees.
- Problems that show value of AMCS and STEM.
- Steady and significant annual growth.

Needs: "how to" guidebook;
curriculum support;
teacher and math education
development;
standards alignment;
recognition of value.



Michelle Montgomery
Marketing, Outreach, M³ Challenge



American Statistical Association

Promoting the Practice and Profession of Statistics

- Rebecca Nichols (Director of Education)
- ASA is working to enhance statistics education, including statistical problem solving and modelling, at all levels
- Census at School (example project for grades 7-12)
 - International classroom project that engages students in statistical problem solving and modelling using their own real data
 - Students complete a brief online survey, analyze their class census data, and compare their class with random samples of participating students in the United States and other countries
 - Over 18,000 students in 45 states plus DC have participated
 - Lessons and resources tied to Common Core State Standards and ASA's GAISE Pre-K-12 Curriculum Framework
 - www.amstat.org/censusatschool
- Needs
 - Provide students with more experiences modelling with real data, real-life applications, and career opportunities



Oana Pascu – MFA Master Teacher

SIAM-NSF Workshop on Modeling Across the Curriculum

Cornell Mathematical Contest in Modeling (CMCM)

- Teams of three undergraduates
- Address one of two "real-world" problems
- Students:
 - build a mathematical model
 - propose solution based on model
 - produce a paper
- Solutions evaluated on feasibility, validation of the model, clarity of presentation.



Works well:

- Set of materials
- Student experience with Modeling
- Similarities to M³
- Raise awareness

Need:

- Supporting courses



Middle School Mathematics

1. Working well: The Connected Mathematics Project (CMP)

- This is a mathematics curriculum for grades 6-8. Development began in 1991 with NSF funding. Now in its third edition.
- Almost every lesson begins with a real-world context which the students are expected to analyze and make sense of. Mathematical learning is motivated by the need to describe, predict, and create in realistic contexts.

2. A critical need: More research into how students learn estimation and approximation.

- Many K-12 students and teachers have primarily been exposed to mathematics where there is an exact answer, and they are not proficient with concepts relating to error and approximation. Realistic examples are sometimes avoided in math classes as a result.
- There is typically not much mathematical language for approximation developed in grades K-8.
- Understanding approximation and "how close is close enough" is particularly critical in middle school, because of the standard mathematical content at those grade levels (for example, ratios and statistics).

Ted Stanford
Department of Mathematical Sciences
New Mexico State University
stanfordt@nmsu.edu



MAC II

Katie Fowler

Dept. Mathematics, Clarkson University

- What is working well: Potsdam High School Senior Capstone Projects (not perfect but successful so far)
 - Applications!
 - "local" problems being investigated
 - Support from community
- Critical need for MAC:
 - Communicating and getting support from teachers and parents
 - Easy to criticize/blame "the system" if children are challenged
 - Unaware of pay-offs/benefits
 - Not in their comfort zone
 - Lack of communication/clearly defined goals and outcomes
 - Need a unified commitment to the changes in practice!

Grades 7 - 12

What's Working:

- Existing program:
 - *STEM
 - *Common Core Curriculum
 - The 8 Mathematical Practices
- Materials
 - *Geogebra
 - *STEM for all lessons – although these could be improved from a math stand point (I'll bring some in that our district worked on last year).

Critical Needs:

- Consistency amongst teachers – teachers don't have a requirement when it comes to modeling across the curriculum
- Teacher training – I've never had a PD session or any courses that have focused on modeling
- Expand – not just focus on modeling within the STEM courses, but applying it to all levels and different types of students. All students benefit from modeling.

Name: Lauren Leischer
Affiliation: 7th Grade Math Teacher

Appendix C Introductory Slides

UNIVERSITY OF DELAWARE

Level: Grades 7-12
John A. Pelesko
 University of Delaware
 Department of Mathematical Sciences

Existing Training Methodology – Delaware

Focus: Training teachers and teacher leaders in the state to be competent mathematical modelers.

Key assumption: To effectively teach modeling, teachers must first be competent modelers themselves.

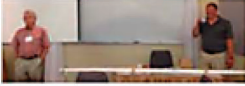
Key component of methodology: Deep working partnership between mathematicians and mathematics educators (i.e., someone who does modeling working in partnership with a mathematics educator who designs professional development).

Critical Needs:

Expanded opportunities for teachers to learn to be mathematical modelers. Modeling camps for teachers.

Better integration of mathematical modeling into teacher education programs at all levels.

Strong participation by mathematical modelers in SBAC and PARC item design.



Undergraduate Sustainability Experiences in Mathematics

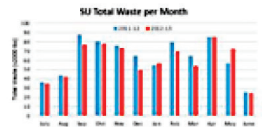
USE Math
 7-12/Introductory College

NSF-funded project to develop relevant, reachable, and reflective sustainability-focused activities.

Available online: http://serc.carleton.edu/sisl/sustain_in_math.html

Needs:

- Review
- Assessment
- Alignment with CCSS



Ben Galluzzo
 Shippensburg University

Undergraduate Curricula

Modeling Across the Curriculum Undergraduate

First Semester of Calculus: Karen Saxe, Macalester College

Part 1: Constructing models

- Unit 1 - Functions as models & parameter fitting
- Unit 2 - Units, dimensions
- Unit 3 - Linear algebra & model fitting

Part 2: Using models

- Unit 4 - Concepts of derivatives
- Unit 5 - Symbolic differentiation
- Unit 6 - Optimization
- Unit 7 - Integration & accumulation

Part 3: Models of change

- Unit 8 - Differential equations

Our Goals? Design good terminal course for science majors. Expose them to linear algebra and multivariate differentiation in first semester.

What works well? Is prerequisite for Stat. Modeling & 2-course sequence is successful. Heavy use of R, almost exclusively applied problems, long-term projects.

Challenges? Placement in calculus stream from HS, and transition to subsequent (& traditional) calculus course. Next fall the dept. is reorganizing calc II and calc III along similar lines, the new courses will be called Applied Multivariable Calculus I, II, and III (and will be our only calculus classes). Placement will still be a challenge, for AP/IB students.

What if modeling is an "end" rather than a "means" (at College/University)?
 Simon Tavaroz, Colorado State University

I - Existing programs	II - Critical needs
Graduate Programs in Mathematical Modeling My questions: - Who is the audience? - What is "mathematical modeling"? - What are the objectives? Birmingham Mathematical Modeling Inc. Lincoln: NSF in Mathematical Modeling Rosen: Mathematical analysis and modeling - Students from a strong mathematics background seeking to become generally multidisciplinary scientists. - Topics: Analysis, Continuum mechanics, Dynamical systems, Game theory, Numerical Analysis, ODE, Optimization, PDE, Perturbation theory, Probability, Statistical inference, Stochastic processes. - Applications: Mathematical biology and physiology, Ocean sciences, Finance. - Train students for careers in scientific research, and for employment in a wide range of industrial contexts, for example biotechnology, industrial engineering or the pharmaceutical industry.	- Are there equivalent programs on the side of the Atlantic? Do we need some, more? - My recommendations if so: - MS level - Adapt to local mathematical expertise - Adapt to local industrial and professional environment - Collaborative effort between mathematics and statistics - Are there programs aimed at non-mathematicians? Do we need them? What is feasible/desirable? - Humboldt State - "Mathematical modeling" is a concentration within the Environmental Systems Graduate Program. - Example: The Plant Cell (a publication of ASPB) initiated a feature called "Teaching Tools in Plant Biology" (TTPB) in 2009 to: - Support plant scientists across the globe in their undergraduate and graduate-level teaching - Provide up-to-date content and bibliographies, images, and connections between basic research and the applications of plant science - In 2012, TTPB received more than 315,000 page views, originating from 104 countries

Mathematics for the Life Sciences: College Entry-Level

- A 2-semester sequence that incorporates the mathematical concepts that are critical for modern biology, develops these in novel ways closely linked to biological observation and theory, and includes a computational emphasis. Modeling, descriptive statistics, matrix algebra, probability, discrete dynamical systems, basic calculus, differential equations, emphasizing data and hypothesis formulation. **Math for the Life Sciences** (Princeton U. Press, 2014) Bodine, Lenhart and Gross.
- Challenges: Incorporating quantitative methods throughout the curriculum. Expanding appropriate use of computational tools at all levels. Building links to biological data analysis and more advanced modeling through research experiences.



Erin Bodine



Suzanne Lenhart

Louis Gross, University of Tennessee, Knoxville

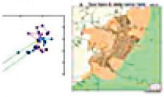


Appendix C Introductory Slides

Applied and Industrial Mathematics (AIM) at Virginia Military Institute
John David



Optimal Meal Delivery Routing
Sponsored by Wiley Program for the Aging



Food Price Prediction
Sponsored by VMI and YoungLife



Detecting Pirates
Sponsored by Johns Hopkins Applied Physics Laboratory



City Energy Database
Sponsored by City of Lexington



We need to expose our students to as many real problems as possible

Mathematical modeling modules have been successful in the past (e.g. COMAP's UMAP and HIMAP Modules).

New modules that reflect new mathematical tools and new areas of applications should be written and made available for teacher and student use.

Joseph Malkevitch (York College and Graduate Center (CUNY)) (Particularly interested in college and high school level mathematical modeling.)

College/University Level – Giampiero Campa

What works well:

- Adopt a single computational tool systematically across the curriculum: This allow students to develop confidence and consistent expertise, and to focus on the concepts/problems, without having to relearn the tools.
- Engineering: (e.g. Berkeley, Stanford, M. Tech, many others)
 - Programming, Intro to Eng., Lin. Algebra, Numerical Analysis, Heat/Thermo, Circuits, ODE, Dynamic Systems, Control, Design Classes
- Physics: (e.g. Siena College)
 - Programming, Mechanics, Electromagnetism, Waves, Optics, Computational Physics, Linear Algebra, ODEs, Astrophysics, Senior Classes/Projects
- Math: (e.g. Oxford, MMU)
 - Symbolic Calculus, Programming, Vectors, Linear Algebra, Numerical Methods, ODEs, Dynamic Systems, Chaos, Optional Applied Math Courses (Finance, machine Learning ...)

What is still needed:

- Positioning math as an essential tool for describing problems
- More interactivity, more cycles Concepts->Practice->Problems
- How to chose the right model in a hierarchy (simple->complex->reality)

Lizette Zietsman, Virginia Tech.

Programs/courses at VT:

1. First year course that includes linear algebra, multivariable calculus and differential equations. Real world applications are an integral part of the course.

Remarks:

- ▶ Advantage to introduce modeling early in student's career.
- ▶ This approach often does not fit in traditional curricula.

2. Senior level applied mathematical modeling course. Students choose real world applications of interest. Recent projects include ranking of sports teams and scheduling of our local bus system.

Remarks:

- ▶ Often student's first exposure to modeling.

3. New degree program in Computational Modeling and Data Analytics, Fall 2014.

Appendix C Introductory Slides

Robin Lock, St. Lawrence University, College

Stat2 : Models for a World of Data
(a course to follow intro stat)

Response

=

Model

+

Error

Quantitative or Categorical

↑

Multiple variables Quantitative or Categorical

↑

Residual Analysis to check conditions and improve model

↑

CHOOSE – a form for the model
FIT - estimate coefficients
ASSESS – conditions and fit
USE – answer question of interest

Challenge: Finding good, interesting data situations and questions

Kelly Black, Clarkson University

Calc/Physics & Linearity (UNH)
 Complete Program
 Calculus, Physics, Linear Algebra, MV Calculus, and ODEs
 Modeling & analysis integrated through courses
 Modeling combines topics and moves ideas forward.

Needs for future:
 Willingness to rethink entire curriculum
 Build connections across a variety of departments
 Consistency across multiple courses and departments.
 Statistics!

This should not be treated as a separate course.

www.distributome.org

An open-source resource for teaching (mostly) univariate probability models. As examples - properties of and relationships between distributions and simulators and calculators and games and activities.

Dennis Pearl, OSU

Modeling Across the Curriculum II
 (January 12-14, 2014)

Level: Undergraduate/College
Successful Program: (i) Mathematical Problems in Industry summer workshops (at RPI, WPI and UDEL); (ii) Harvey Mudd College Mathematics Clinic
Critical Needs: **Availability** of curricular materials in applied mathematics suitable for use by non-applied mathematicians. Materials should include examples rooted in real-world industrial mathematics problems that sophomore-level (no advanced mathematics training) can understand and appreciate.

Ron Buckmire, Occidental College, Los Angeles, CA, ron@oxy.edu

Appendix D Exemplars of Mathematical Modeling Tasks for the Early Grades

A. Traditional Classifications of Learning Tasks

As discussed above, many teachers misunderstand the term “mathematical modeling”. Although modeling appears in the CCSSM, the word modeling in math education is most commonly used in two ways: 1) the act of the teacher demonstrating how to execute a procedure or process, and 2) the act of representing a problem through physical means with manipulatives. Most elementary teachers have virtually no exposure to the mathematical modeling of the STEM fields; they are even less likely to be familiar with the uncharted territory of mathematical modeling tasks in the elementary curriculum. Thus, an important task of the workshop was to search for exemplars that could serve as targets for each of the early grades. However, many of the tasks available, even those described as “Level 4”, “higher level”, or “open-ended” would require some modification to be considered true modeling tasks.

B. Non-examples of Modeling Tasks

Here we show some problems that fall lower on the task taxonomy than possibly intended. We’ll use the modeling taxonomy from above: Bare ► Application ► Model ► Modeling to classify these problems.

1. The following problem was cited as an example of a mathematically rich and open-ended task, but is only an application problem.

<http://mathforum.org/pow/samples/packet4451.pdf>

How Many Berries Did I Eat?

I have a blueberry bush with 9 blueberries and a raspberry bush with 7 raspberries. I ate some blueberries and some raspberries. Now there are 4 blueberries and 4 raspberries on my bushes. Did I eat more blueberries or more raspberries? How do you know?

Check your thinking by using pictures or numbers to show how many blueberries I ate and how many raspberries I ate.

2. The following problem is a sample of the international PISA test. The problem requires estimation (a useful pre-modeling skill) but it still is only an application problem.

Rock Concert

For a rock concert a rectangular field of size 100 m by 50 m was reserved for the audience. The concert was completely sold out and the field was packed with all the fans standing. Which one of the following is likely to be the best estimate of the total number of people attending the concert?

A. 2,000 B. 5,000 C. 20,000 D. 50,000 E. 100,000

While multi-step problems such as these require a concentrated effort in first understanding the problem, reasoning, and higher-level thinking, they nevertheless would need modification to be considered true mathematical modeling tasks. In the next section we will show how such modification can contribute to forming pre-modeling tasks.

Examples of Pre-Modeling Activities

Although many of the problems found in textbooks and even described as higher-level thinking tasks are actually applications, with a little adjustment these can be made into richer tasks. The purpose of these tasks is to have students reason through their choice of solution method and validation, rather than follow memorized procedures. Thus they are making meaning and sense of the problem rather than simply getting an answer.

For example, instead of giving students several numbers and asking them to find the average, one might do the reverse: give students the average and ask them which data might have led to that average, as in the following problem: “After five games, the goalie had averaged blocking six goals per game. What might be the number of goals he blocked in each game?” (Sullivan and Lilburn, 2002, p.4). Instead of asking students to find the perimeter or area of a rectangle, one might give them the perimeter and ask what the area might be, as in this problem: “I want to make a rectangular garden. I have 30 meters of fence to enclose my garden. What might be the area of the garden?” (Sullivan and Lilburn, 2002, p.3) Suney Park posed the following problem to her sixth graders (<https://vimeo.com/46127286>): “There are 22 guests at a table. Each guest’s place is one yard long at the table. What are the dimensions of the table that will allow the most space for food? What are the dimensions of the table that will leave the least amount of room for food while still seating 22 guests?”

An extension that borders on modeling would be to ask students to generalize their findings for any number of guests.

A similar example on the topic of fractions and percentages leads students to examine patterns and understand the underlying concept more fully: “In a survey I found that $\frac{3}{4}$ of the people liked Kobe Bryant. How many people did I ask, and how many like Kobe?” (Adapted from Sullivan and Lilburn, 2002, p.15) Open-ended questions such as these can lead to more discussion and a deeper understanding of the underlying concepts than students would gain by simply a memorizing a procedure.

When students are challenged to construct a piece of mathematics theory for themselves, as opposed to merely applying what they were just taught, everyone benefits. Meyer (2012) posed a problem of the week to his high school seniors, which always sparked more interest than any other task. Students were engaged in the practice of doing mathematics and discovering for themselves. In fact, they communicated to him a wish for a class curriculum composed entirely of “problems of the week”. He explains, “The problem is pitched like a puzzle. There is a clear question, but the solution to that question is not the end of the problem. The problem ends

Appendix D Exemplars of Mathematical Modeling Tasks for the Early Grades

(potentially) when students create a piece of mathematics to describe what they are noticing.” More than simply asking students to obtain the answer to a specific case; he asks them to generalize or create a rule, which will form longer-lasting learning. For example:

A square 6x6 milk crate can hold 36 bottles of milk. Can you arrange 14 bottles in the crate so that each row and column has an even number of bottles? Examine crates of other sizes. Can you create a rule that will tell us which numbers of bottles will be possible with these constraints for ANY size crate? (Meyer, 2012)

One way we might increase the level of difficulty of word problems to present the question without any of the information that typically accompanies it. For example, “How many tables will we need to set up for the assembly?” Or, “How many buses will we need for our field trip?” This leads students to generalize and to think more realistically about quantities rather than rushing to choose an operation and insert the given numbers into that operation. It is important for students to determine what information they need in order to answer the question and where they might locate that information. Because this requires students to research and to differentiate between important and unimportant information, this type of task would help prepare students for other modeling tasks they will encounter.

Another way to start from a textbook problem and invoke higher-level learning is to ask students to compare and contrast problems of different types. Van Dooren (2011) found that participants who were given a set of word problems to classify (but not solve), followed by a different set of word problems to solve, did better at solving them than the group that had not done the classification activity prior. Interestingly, the group that solved a set of problems first, followed by a classification activity on a different set of problems second, actually performed worse at the classification activity than the other group. Thus it is valuable for students to analyze commonalities and differences rather than focus on getting the right answer. If students have practice identifying the situation represented by a problem, as opposed to just executing an operation, they might be more able to represent a real-world situation mathematically without guidance or specific instruction from the teacher, one essential component of modeling.

Also essential in preparing students for modeling is to encourage students to draw upon rather than reject their prior knowledge about the world. To this end, routine problems can be made non-routine by asking students to determine how real-world considerations might affect an otherwise simple mathematical procedure. For example, the following at first seems to be a standard textbook problem involving distance, rate, and time, but further discussion can spark creativity and lead to significant decision-making:

Grandmother will arrive at the airport at 6:00pm. The airport is 20 miles away, and the speed limit is 40mph. When should you leave for the airport? What are other considerations? (Pollak, cited in Engage NY, 2013)

A discussion of this problem with students might involve them in the type of thinking mathematical modelers face: depending on what factors they choose to take into account, they can approximate the most fitting departure time with greater accuracy. This is the kind of decision-making -- which assumptions to make; which information to consider and which to deliberately ignore -- that is present in the modeling process.

We should not underestimate the amount of mathematical knowledge and ability to make sense of problems that students bring with them. It is rather too often the case that we reduce the cognitive difficulty of the task below that of which students are capable.



Notable studies (Verschaffel 2010, and Greer 1997) show that students appear to suspend sense-making when solving word problems. For example, given the problem: “A shepherd has 23 goats and 10 sheep. How old is the shepherd?” most students answered 33, which seems to be irrational. However, during videotaped interviews with students, many actually laughed or made a noise of surprise when asked this question, though they still answered 33. Students still attempted to make sense of the problem when they defended their answer. For example, one student justified his response by saying that perhaps the shepherd received one animal for each year of his life.

Thus it could be argued that students are acting as highly rational beings: in the classroom world, which is part of the real world, textbook problems usually contain whole numbers that are divisible by each other, and the operation students need to use is usually the one that was just taught that day. In one study, teachers were even shown to reward non-sense-making answers more highly than sense-making ones that took into account more real-world considerations (Greer 1997). We need to give students more credit.

Appendix D Exemplars of Mathematical Modeling Tasks for the Early Grades

They have in fact recognized a pattern in their schoolwork, and that is that suspending their sense-making in a classroom situation effectively and most frequently leads them to get the “right answer.” Therefore contrary to appearances, by ignoring their real-world knowledge when answering textbook problems students are in fact acting as highly rational individuals. This is further reason to take learning beyond traditional textbook problems. Part of training students to be modelers is not to train their sense-making capabilities out of them! This is yet another reason to begin modeling in the early grades and to teach it in every grade.

A valuable way to take advantage of students’ natural ability and interest in making sense of problems, while raising the level of cognitive difficulty, is to present them with problems that would not typically be presented at that age level. For example, calculus is one way to solve optimization problems, but that does not mean that middle schoolers cannot understand those problems and reach a close estimate through trial and error or graphing. Similarly, multiplication and division problems are usually reserved for 3rd grade and up, but CGI research has shown kindergartners with no prior instruction on multiplication or division being able to independently model a scenario requiring those operations (Carpenter, et al, 1993). By using direct modeling (representing with manipulatives) half the children were able to correctly solve the following problem:

19 children are taking a mini-bus to the zoo. They will have to sit either 2 or 3 to a seat. The bus has 7 seats. How many children will have to sit three to a seat, and how many can sit two to a seat?

One could argue that future grades’ learning “undoes” children’s natural inclination and ability to make sense of the problem. Often math instruction trains students to look for the operation based on key words so that they might immediately perform that operation with the numbers in the problem.

We should not underestimate the ability of students to make sense of unfamiliar problems. As another example of this, Brown (add citation) gave fourth graders a typical sixth grade proportional reasoning problem involving two wheels revolving at different rates. These students had not been introduced to proportions but creatively and successfully used pictures, tables, and logical reasoning to arrive at the solution. Moreover, they were able to articulate their reasoning process clearly. On the other hand, the sixth graders who were given the same problem uniformly solved it by setting up a proportion, which they struggled to explain. Thus teaching students always to use a specific strategy for a certain type of problem can be counter-productive. Drawing instead upon students’ natural ability to make sense of their world can result in longer lasting learning and greater confidence.

Although at first the tasks of giving multiplication and division problems to kindergartners and proportional reasoning problems

to fourth graders may not appear to be modeling, these examples show students using models in a way in which they have not been trained but which makes sense to them. Thus there is some creativity and reasoning required, which is part of the modeling process. This creativity and the ability to form representations of unfamiliar relationships are important qualities that we want to develop in modelers; therefore, giving tasks that are “beyond” students’ level of curricular knowledge is a worthwhile pre-modeling activity.

We have discussed the following strategies: starting with the question, asking students to make generalizations, having students determine and research the information they need, brainstorming real-world considerations, analyzing similarities and differences, showing a variety of solution methods from student work [include paragraph on this from Smith and Stein], and presenting problems before the grade level they are traditionally taught. All of these “pre-modeling” activities have the potential to contribute to a strong foundation for mathematical modeling.

C. Examples of Modeling Tasks

To be considered mathematical modeling, a task generally must contain several of the following attributes:

Open-endedness: There might be more than one possible solution. Furthermore, the area of mathematics needed for the solution is not suggested by the problem or by instruction that took place immediately prior.

Problem-posing: Students decide what questions to ask, what they need to research, and how to state the problem in math terms.

Creativity and choices: Students may choose not only the model, but the area of mathematics that might help them solve the problem. In addition, students must make assumptions by determining which factors to take into account and which to ignore. Students must also choose a reasonable level of accuracy.

Iteration and revisions: Students test whether their model gives a close enough approximation and revise it to obtain an even more accurate solution.

We have determined some potential modeling tasks for students in the early grades. It should be noted that many of these tasks may be accessible to a wide span of grade levels; students at different ages may choose different approaches to modeling the situation.

Fish in a tank. (Kinder) What questions could you ask about all the fish in a fish tank? Have students quantify their observations and questions in ways they choose.

Appendix D Exemplars of Mathematical Modeling Tasks for the Early Grades

Paying at the register. (1st-2nd) You are buying a pair of shoes. How much would they cost? What are some ways that you might make that amount in bills? In coins? (Kinch 2011)

Giant's foot problem. (3rd-6th) Given the footprint (or handprint) of a giant, determine how tall the giant was. (involves measurement, proportional reasoning, and statistics.)

<http://nzmaths.co.nz/resource/giant-mystery>

In a similar task, Brown (add citation) investigated the size of polar bears with third graders. Students posed questions such as “Is a polar bear taller than the ceiling?” “How many Ryans (a student) would fit in a polar bear?” They took measurements, researched polar bear specifications, and made posters showing the relative heights of third graders and polar bears.

Fundraiser. (3rd and up) Design (and perhaps implement) a fundraiser that will allow your class to go on a field trip.

Counting trees. (4th-7th) Given a rectangular diagram of a tree farm in which tiny circles show old trees and the triangles show young trees, estimate how many trees there are of each type. (Involves estimating, proportional reasoning, and statistics.) <http://map.mathshell.org/materials/lessons.php?taskid=422&subpage=problem>

Counting Trees

The diagram shows some trees in a tree farm.

The circles • show old trees and the triangles ▲ show young trees.

Tom wants to know how many trees there are of each type, but says it would take too long counting them all, one by one.

Pizza problem. (6th-7th) Which local pizza place offers the best value for pizza? (Involves measurement, ratios, and statistics.)

Euler's Konigsberg bridge problem. (4th and up) What is the route that will allow you to cover all the streets while making the fewest number of overlaps or double trips? (Uses graph theory with applications in snowplowing, street sweeping, mail delivery, etc.)

Best location for a hospital. (6th and up) Suppose you are hired to determine the best location for a shared medical facility [central to three given cities]. (Uses topology, geography, statistics.) (Zbiek & Conner, 2006)

Poster Problems. SERP Institute in San Francisco has been working to develop a series of problems called “Poster Problems” for 6th and 7th grades to help teachers engage in what they call diagnostic teaching. The materials are accompanied by teacher training materials to help clear up typical misconceptions that might be held by teachers or students and bring students’ thinking into the open. Beginning with an engaging question such as “Can a dragonfly fly faster than we can drive?”, students collaborate in groups to come up with solutions on posters. Afterwards, students view and respond to other groups’ posters with adhesive notes and engage in a class discussion.

http://math.serpmedia.org/diagnostic_teaching/

Authentic tasks. An important aspect of industrial mathematics is that there is a client who is not the one modeling, but is invested in the outcome of the model. In the early grades, students can look around their school and try to model something that they see as a problem, which could lead to a proposal for a solution. In this case, the school administrators could be brought in as the clients. Students could also visit local business to identify possible issues that could be better understood through mathematical modeling. Having a real client can be a tremendous motivator for team or individual modeling efforts. It lends the assignment an authenticity similar to that of the vital publishing component in Writer’s Workshop, giving students a taste of the satisfaction that can come from being a modeler.

ACTION ITEM: Create SIAM/ASA websites where teachers can view lesson plans. Include a SIAM library of models that have been vetted by the community and a library of best practices in teaching/learning modeling (including international examples). Include multiple paradigms of modeling.

ACTION ITEM: Create a nationwide modeling challenge for early grades. One problem per year. Encourage students all over the country to try a common problem. Mini Moodys?

D. International Examples

As we work to integrate mathematical modeling in the US, we can learn and exchange ideas with our colleagues in other countries. Finland and Singapore have received much positive attention for their implementation of mathematics curriculum, but we can see that the process of really having students engage in modeling is challenging there as well. Germany and Japan also consistently outperform the U.S. on international tests such as PISA.

Appendix D Exemplars of Mathematical Modeling Tasks for the Early Grades

A. Finland

(Motto: learn by doing)

Erkki Pehkonen, University of Helsinki, Finland How Finns learn mathematics and science (Pehkonen, Ahtee & Lavonen 2007).

(Pehkonen 2008): background information on the development of the Finnish mathematics instruction and curricula within last 30 years.

In the 1990s, responding to the new demand, a group of Finnish mathematics educators wrote a booklet on mathematics teaching (Halinen & al. 1991), presenting a view very similar to the later concept of mathematical literacy in PISA

- two key points arose: understanding learning as an active endeavour, and mathematics as a skill to be used and applied in diverse situations.
- The first project “Open tasks in mathematics” was implemented in the upper grades (grades 7–9) of the comprehensive school in 1989–92 in Helsinki area. In the first research project teachers’ and pupils’ beliefs were recognized as obstacles for change (cf. Hannula & al. 1996).
- The third project “Teachers’ conceptions on open tasks” that was implemented in 1998, concentrated on the second observed obstacle: teachers’ pedagogical knowledge (cf. Vaulamo & Pehkonen 1999).
- Research project “Understanding and Self-Confidence in School Mathematics”, financed 2001–03 by the Academy of Finland.
- Research project “Elementary Teacher Students’ Mathematics”, financed 2003–06 by the Academy of Finland.
- In Finnish mathematics teaching the direction seems to be to more individualizing in the comprehensive school, and mass teaching in the secondary schools.

B. Singapore

Motto: Teach Less, Learn More (TLLM), a call made by PM Lee Hsien Loong in his inaugural National Day Rally speech in 2004. “Teaching will be focused on developing understanding, critical thinking and the ability to ask questions and seek solutions”

http://www.seameo.org/index.php?option=com_content&view=article&id=114&Itemid=531

Singapore Math Curriculum
<http://www.singaporemath.com/>

K.C. Ang (2001) describes the challenge in Singapore as well of presenting genuine problems, which are more likely to be messy and to span disciplines:

“In practice, however, the emphasis has been on solving routine mathematical problems in a context-free environment. Even on the odd occasion when a “real life” problem or example is discussed in the classroom, it is typically a rather artificial problem created for the purpose of fitting it into the topic in question. The problem is usually complete by itself, and is presented in a very clean and tidy state. Such practice makes it difficult to convince the learner that real life applications of mathematics do indeed exist. In addition,

mathematics has often been thought of by pupils as consisting of a set of distinct topics that are compartmentalized and self-sufficient. In real life, however, problems tend to transcend a number of disciplines and are often not so well defined. Often, we need to apply ideas and concepts in one area to solve problems arising in another. Mathematical modelling offers excellent opportunities to connect and use ideas from different areas.”

C. Japan

“Stevenson and Stigler (1992) contrast pedagogical practices between the typical U.S. teacher and teachers in Asian countries such as Japan and China. Key to this comparison were some of the misconceptions that Americans have about a rote instructional approach to instruction in the form of drill and kill exercises (Stevenson & Stigler, 1992). Teachers in the countries mentioned above were much more apt to pose challenging questions to students and provide them opportunities to reason through the problems (Stevenson & Stigler, 1992). Math problem-solving is an area of concern around the mathematics achievement of U.S. students. It was found that U.S. students fell furthest behind on PISA tasks that required complex problem-solving (Darling-Hammond, 2010). Differences in approaches to math instruction consistently point to the observation that nations who significantly outperform the United States on math achievement have classrooms characterized by a focus on mathematical reasoning and problem-solving with students interacting with real-world problems (Darling-Hammond, 2010; Stevenson & Stigler, 1992). The emphasis is on fewer problems with more depth of understanding where collaborative work on one problem could very well take the whole class period (Darling-Hammond, 2010; Stevenson & Stigler, 1992).

“Stigler and Hiebert (1999) focused specifically on such differences between 8th grade math teachers in Japan, Germany, and the United States. They constructed three distinct mottoes to characterize the norm of pedagogical practices in each country as follows. The motto for Japan’s general approach to math teaching was ‘structured problem-solving’ characterized by posing demanding problems with students taking an active role in inventing their own solution strategies (Stigler & Hiebert, 1999, p. 27). The motto attributed to Germany’s math instruction was ‘developing advanced procedures’ characterized by advanced procedural problems and technical precision with applying these procedures (Stigler & Hiebert, 1999, p. 27). Finally, the motto for United States mathematics instruction was classified as ‘learning terms and practicing procedures’ characterized by less advanced problems with less demands for mathematical reasoning (Stigler & Hiebert, 1999, p. 27).

“Japan’s motto for math instruction was classified as ‘structured problem solving’ (Stigler & Hiebert, 1999). In a typical lesson in a Japanese classroom, it was common for students to present multiple solution strategies to a problem allowing for students to learn from one another. Furthermore, any errors in reasoning were not instantly corrected by the teacher, as is the case in typical U.S. math instruction. Mistakes in Japanese lessons were an essential part of the learning process (Stigler & Hiebert, 1999, p. 91). Our culture of avoiding errors in practice contradicts the very nature of learning. Trial and error, multiple revisions in thinking and work are an inherent part of human learning processes.

Appendix E

Assessment of Mathematical Modeling Activities

The mathematics curriculum has several aspects: the stated curriculum (say the CCSS), the enacted curriculum (what happens in the classroom) and the assessed curriculum (such as what is tested on standardized tests). It is easy to envision how there could be disconnects between these different notions of curriculum. The enacted curriculum can be affected not only by the teacher's strengths and preferences, but also by both the stated and assessed curriculum.

While we may not think of Mathematics Education experts as typical SIAM members, we can work with Mathematics Education researchers, curriculum developers, teacher trainers and teachers to develop creative and illuminating ways to assess mathematical modeling skills. We have some examples of rubrics for evaluating mathematical models from the COMAP Mathematical Contest in Modeling (MCM) [2] and Interdisciplinary Contest in Modeling (ICM) and Moody's Mega Math Challenge [3] modeling competition. While the competition papers are usually evaluated as a team effort, in the classroom setting teachers might need to be able to assess the efforts of individuals. Industrial mathematics programs and modeling courses at the high school and undergraduate level can provide models of individual assessment for group modeling projects.

Mathematics proficiency in 45 states will be assessed using instruments developed by two testing organizations, SBAC, the Smarter Balanced Assessment Consortium [4] and PARCC, the Partnership for Assessment of Readiness for College and Careers [5]. How these agencies decide to assess modeling and the types of example problems that they release will likely have a tremendous effect on what teachers do in the classroom. We hope that SIAM members with expertise in modeling and education can work with these agencies to develop modeling assessment tasks.

The following are some examples of released performance tasks from SBAC.

Grade 6 field trip

<http://www.smarterbalanced.org/wordpress/wp-content/uploads/2012/09/performance-tasks/fieldtrip.pdf>

Your class and your teacher are going on a field trip. There are three possible choices for the field trip: an aquarium, a science museum, or a zoo. Your teacher asked students to write down their first and second choices. In this task, you will determine where the class should go on the field trip based on the survey results and the cost per student.

Grade 8 Design a Park Smarter Balanced Grade 8 Level 4 task.

<http://dese.mo.gov/divimprove/assess/documents/asmt-sbac-math-gr8-sample-items.pdf>

During the task, the student assumes the role of an architect who is responsible for designing the best plan for a park with area and financial restraints. The student completes tasks in which he/she compares the costs of different bids, determines what facilities

should be given priority in the park, and then develops a scale drawing of the best design for the park and an explanation of the choices made. This investigation is done in class using a calculator, an applet to construct the scale drawing, and a spreadsheet.

or (in PowerPoint form)

<http://www.smarterbalanced.org/wordpress/wp-content/uploads/2012/02/Showcase%203%20Webinar.pdf>

Additionally, here are some items from PARCC.

Grade 3

An art teacher will tile a section of the wall with painted tiles made by students in three art classes. Class A made 18 tiles. Class B made 14 tiles. Class C made 16 tiles. What is the total number of tiles that are to be used? The grid shows how much wall space the art teacher can use. Use the grid to create a rectangular array showing how the art teacher might arrange the tiles on the wall. Select the boxes to shade them. Each tile should be shown by one shaded box.

http://www.parcconline.org/sites/parcc/files/ArtTeacherRectangularArray_o.pdf

Grade 4

Ms. Morales has a bag of beads. She gives Elena 5 beads. She gives Damian 8 more beads than Elena. She gives Trish 4 times as many beads as Damian. Ms. Morales then has 10 beads left in the bag. How many beads did Damian and Trish each receive? Show or explain how you arrived at each answer. How many beads were in Ms. Morales' bag before any beads were given to students?

<http://www.parcconline.org/sites/parcc/files/Grade4-ThreeFriends%27Beads.pdf>

ACTION ITEM: Work with both testing agencies to devise mathematical modeling problems and assessment rubrics to help communicate what types of modeling activities teachers can facilitate in the classrooms.

Appendix F Outlines for High School Modeling: Proposals and Recommendations (AIM-style workshop proposal)

Planning Workshop Meeting (AIM or SIAM New Initiatives): Preliminary thoughts on the content and structure of such a proposal

The introduction should include thoughtful reference to the PCAST, 2025 and other reports. The topics that are in the common core, for example, can all be taught in the context of modeling; we can use this idea to frame out what the workshop should achieve. Decide on a structure: what does a typical day in the workshop week look like? (e.g. daily summary of what's been done, where we're going; then divide into smaller groups & work; recombine) All strands should be based on/linked to research literature.

- Modeling across the curriculum tasks: part of this strand should address the variation in structure of effective modeling courses or other pedagogical choices.
 - Course development: develop a prototype modeling course (courses or a suite of 2-3 courses that address different teaching choices/contexts) that many schools can aspire to teach, based on existing successful courses at the HS level. Part of this address research-based philosophy in course development - what are the guiding ideas in terms of teaching modeling)
 - Infusion development: make recommendations for approaches to infusing modeling thinking/approaches across the secondary curriculum with topics from STEM and non-STEM disciplines
- PD development: what PD activities exist or should be developed in to support the coursework and other infusion activities for both pre-service and in-service teachers; can PD choices be guided by the CUPM report (consider levels of PD frameworks to reach the greatest number of teachers: e.g., 1 hour, 2 hour, 10 hour)
- Assessment development (assessment of modeling skills across K-16): what assessments should be developed or used that support these efforts

In writing the proposal here are guiding principles for each theme:

1. What's been done/summarize best practices
2. Identify gaps and challenges
3. Propose ways to transform mathematics education culture at a large scale

Appendix F Outlines for High School Modeling: Proposals and Recommendations (AIM-style workshop proposal)

Proposal for a GAISE-inspired report, GAIMME: Guidelines for Assessment and Instruction in Mathematical Modeling Education

What this report should accomplish?

GAIMME should reinforce the four “C’s” in the classroom context for mathematical modeling by providing opportunities for students to build skills in Communication, Collaboration, Critical Thinking, and Creativity. GAIMME should provide a gold-standard description of mathematical modeling as well as then setting the stage for curriculum development, assessment development, and professional development at the all levels of the educational pipeline. GAIMME should provide examples of modeling practice and assessment within the K-16 curriculum. GAIMME will provide a blueprint for how mathematical modeling should be effectively taught in preK-16 and beyond.

Why should there be such a report?

Despite high public awareness of the “importance” of the mathematical sciences in developing technologies that have become integral to the functioning of our social and commercial infrastructures, many people, political leaders, citizens, parents, even teachers often do not have a clear understanding of what mathematical modeling really entails (even though they do it all the time). The mathematical sciences community and the education community have the expertise to communicate, particularly with parents and guardians and school leaders, the importance and nature of mathematical modeling for student success in school and beyond. Working on interdisciplinary problems is the future of today’s students, and meaningful mathematical literacy for the 21st century is only the beginning. The 2025 Report emphasizes the critical connections between mathematical modeling and real world experiences and careers. Examples include Secure Internet Commerce (number theory, prime numbers); Satellite Tracking and Video Games (quaternions); Internet Search Engines such as Google’s PageRank (linear algebra, eigenvectors); MRI and PET scans (integral geometry); and many others from medicine, business, economics, environmental sciences, and more.

Despite its integral use in many classes and disciplines throughout the STEM curriculum, mathematical modeling is not explicitly understood as part of the work of STEM studies and research, particularly by students.

Many researchers and educators across the nation and the world have created courses and other kinds of student experiences that foster student understanding of mathematical modeling. Often these admirable and effective works are lost when the person who developed them and supported student learning retires or leaves the profession. A GAIMME report will provide direction for the modeling community to work in concert so that great work is not lost.

A national emphasis on mathematical modeling is a natural continuation and expansion of the historical development of mathematical and statistical science education – modeling is not the next new thing, but a set of skills and habits of thinking that underpin most modern advances in STEM.

From the GAISE initial proposal is an ASA statement about the importance of the project– we argue that a similar scenario exists now for math modeling:

“Currently there are no ASA-endorsed guidelines for statistics education. The 2000 NCTM Guidelines for K-12 are an excellent beginning, but too often they focus on skills and procedures and not enough attention is paid to developing statistical reasoning and thinking or to helping students understand the big ideas of statistics. At the undergraduate level, there have been many articles written and recommendations made regarding teaching first courses in statistics, but there is no specific set of guidelines that can be used to guide development and evaluation of these courses. We agree with the presidents of ASA and the recommendations of the ASA Advisory Committee on Teacher Enhancement that it is ASA’s role to develop and disseminate these guidelines. We are proposing a plan to carry out this important task.”

Who should be on the writing team for this report?

GAIMME requires expertise from K-12 classroom teachers, expertise from teacher educators, expertise from the education research community, and expertise from practicing mathematical modelers. The statistical and mathematical communities both are essential to such a report.

It requires involvement from the whole mathematical professional community certainly including the ASA, SIAM, INFORMS, NCTM, MAA, AMS, AMATYC and others.

MODELING across the CURRICULUM II

Report on the Second SIAM-NSF Workshop*
Alexandria, Virginia
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